A New Formulation for the Geometric Layout Optimisation of Flat Slab Floor Systems

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ABSTRACT

This paper presents a heuristic approach for the optimum layout design of multi-span flat slab floors in order to optimise the total cost. For this purpose, a heuristic methodology is developed in order to achieve a new objective function for the structural optimisation problem. The proposed objective function has the capability to make use of action effects of the structure as the alternative design variables in place of the commonly used cross-sectional ones. Such a feature provides the method with the ability to be easily employed in large and realistic structural optimisation problems. Furthermore, the proposed formulation, in an iterative optimisation process, takes less time than its counterparts as it uses the action effects that can be easily obtained from a structural analysis procedure instead of the cross-sectional variables that are typically the results of a design procedure.

KEYWORDS

Optimisation, Layout optimisation, Flat slabs, Reinforced concrete.

INTRODUCTION

When a structural concrete slab is supported directly by columns without intermediate beams or grids, it is referred to as a flat slab. It is frequently necessary to take advantage of thickened portions, called drop panels, in critical regions close to the supporting columns in order to provide adequate strength in shear. Flat slabs provide flexibility for partition location and allow passing and fixing services easily. Moreover, where the total height of a building is restricted, using a flat slab will result in more stories accommodated within the set height (Warner et al., 1998).

During the recent decades, considerable progress has been achieved in the area of the optimum design of RC structures and many papers have been published in this regard. However, the published works on optimisation of RC slabs have limitations. Some papers deal with reinforcement bars only and others deal with concrete only. Some are limited for certain support conditions of slabs, or limited to academic examples without considering the constraints of an actual design code. Some authors formulate the optimisation problem in terms of one variable only. Some authors do not consider any formal mathematical optimisation at all. Those that do present mathematical optimisation, treat the variable(s) continuous. In other words, only a small fraction of these papers deal with the cost optimisation of large concrete structures and the majority of them only consider the optimisation of isolated elements or simple structures.

There are a limited number of studies in the field of cost optimisation of flat slab buildings. MacRae and Cohn (1987) presented the optimisation of pre-stressed concrete flat slabs based on the Canadian standard for concrete structures. Sahab et al. (2005) considered the cost and topological optimisation of flat slab buildings using a three level optimisation approach. In this study, finding the optimum number of equal spans together with the cost minimization is considered for flat slab buildings. Recently, using new heuristics, some methods have been employed for the layout optimisation of structures by Nimtawat and Nanakron (2009, 2010), Zhu and Zhang (2010) and Shaw et al. (2008).

The problem of optimisation that is put forward in the present work consists of an economic optimisation of a structural design. In this study, a new cost function is proposed to deal with the cost optimisation problem of flat slabs, which can be used in layout optimisation of multi-span flat slab buildings as well as considering the cross-sectional action effects. In order to formulate the structural optimisation problem for realistic structures and subject to actual constraints, the formulation needs to be made based on a design standard. This study makes use of the relations in the Australian standards for concrete structures (AS3600, 2009), which is based on the limit state design method of concrete structures. The proposed formulation results in a constrained nonlinear structural optimisation problem and can be dealt with by various methods to be solved. That is, unlike the commonly used cost optimisation methods, in this method, there is no need to deal with both the design and analysis procedure in each step of the solution. The variables of such a formulation are the action effects that can be easily obtained from a structural analysis procedure instead of the cross-sectional variables that are typically the results of a design procedure.

PROBLEM DEFINITION

Depending on the design problem, four classes of design variables are faced by structural designers: material design variables such as the type of concrete, topological variables such as the number of members in a structure, geometric layout variables such as the lengths of spans, and cross-sectional variables such as the dimensions of sections. Mainly, the existing circumstances of the problem dictate the designer how to set the predefined parameters and design variables. Exploiting the mathematical relationships between design parameters, one may be able to shift from one set of variables to another. In an optimisation problem, such a transition leads to a new definition for the objective function and constraints and may result in the variation of the nature of parameters from design variables to behaviour ones and vice a versa. In other words, depending on the nature of the optimisation problem, the process of achieving an optimum feasible solution may be much quicker, shifting from one design space to another by changing design variables as the space dimensions.

In an optimisation procedure, the definition of the cost function might be considered the most important decision, which presents the aim of the problem. Therefore, it is essential to introduce a cost function that represents the most influential cost components and more importantly, is applicable to a variety of similar optimisation problems. Furthermore, it must be capable of matching the explicit constraints of structures, which are often given by formulas in design standards. A cost function generally includes the cost of materials, transportation, fabrication and even maintenance costs, in addition to repair and insurance costs, which can be presented by a weighted sum of a number of properties. The effect of these factors in optimal cost can be imposed on the weighted coefficients of the cost function. In concrete structures, at least three different cost items should be considered in optimisation: costs of concrete, steel, and the formwork. So, in most former studies the general cost function for reinforced concrete slabs is expressed in the following form:

$$C = c_c A_c + c_s A_s + c_f P_f \tag{1}$$

where c_c, c_s and c_f are the unit costs of concrete, steel and formwork respectively and A_c, A_s and P_f are their corresponding quantities.

The cost function presented by Eq. (1) deals only with cross-sectional variables which mainly suits structures with a small number of members and predefined geometric layout. In fact, in layout design of structures, the cross-sectional variables are functions of design action effects which are not determinate and vary as the shape changes. Therefore, in an iterative procedure to solve an optimisation problem, each step includes dealing with both the structural analysis and structural design variables. In such cases, unless alternative design variables are selected for the cost function, the optimisation procedure might be too unwieldy. That is while parameters like cross-sectional variables are mainly obtained from implicit functions of structural analysis outputs based on the suggested relations and constraints in the design standards. On the other hand design standards do not uniquely provide the exact values for these cross-sectional parameters and they are not obtained from an explicit mathematical procedure. If the reciprocal relationships between the cross-sectional design factors and the design action effects are determined the cost function can be presented by a function of design action effects. In the design process, design action effects are determined for the critical sections of members and then cross-sectional variables are calculated for each section. Cross-sectional parameters along the structural members are obtained from their value in critical sections and based on the relations in the design standards. In fact, having the design action effects in critical sections, all the cross-sectional parameters and consequently, the total cost based on Eq. (1) can be calculated. To put it simply, using structural analysis outputs, say internal actions of a member, as design variables has some advantages over using structural design outcomes such as cross-sectional characteristics of a beam. Firstly, design action effects of each section can be easily obtained from structural analysis, and in an iterative mathematical procedure, re-analysing a structure is considerably less time-consuming and more precise than re-designing the structure. Moreover, using action effects, the cost function will be considered in a section rather than a member. It enables the designer to select a number of sections for each member and in the whole structure to control the cost, and there is no necessity to conduct the optimisation process over the entire member.

Therefore the aim is to explore the relations between the variations of action effects of RC members with the variations of the cross-sectional parameters and find out how these two types of variables affect each other. Then, based on such relations, a new cost function is obtained which is a function of action effects rather than cross-sectional parameters. In order to make the structural optimisation formulation practical, and to impose actual constraints, the formulation is made based on the Australian standards for concrete structures (AS3600, 2009). As a vastly used method, AS3600 (2009) uses the equivalent frame method for rectangular form buildings. In the equivalent frame method the slab of a building is divided into middle strips and column strips for analysis planes in *x* and *y* directions. Figure 1 illustrates how the middle and column strips are defined in a slab. The moments in the column strip frames are calculated using the moment distribution method. The equivalent frame method assumes the moments to be uniform across the strips. The design bending moment of each section of strips in either direction *x* or *y* is obtained from Eq. (2).

$$\begin{cases} M_{ox} \cong \frac{1}{8}\ \beta F_d\ L_{tx}\ L_{oy}^2 \\ M_{oy} \cong \frac{1}{8}\ \beta F_d\ L_{ty}\ L_{ox}^2 \end{cases} \tag{2}$$

where L_t and L_o are the strip widths and effective span lengths in the *x* and *y* directions respectively, and β is a coefficient that indicates the distribution of design strip moments in the longitudinal and transverse directions based on the location of each slab on the floor plan and the position of the section on the strip. The parameter F_d is the factored design load per unit that is the sum of the dead loads and live loads. Therefore, F_d might be defined as a function of the slab depth. The designed reinforcement of a strip section is required to resist the above bending moments.

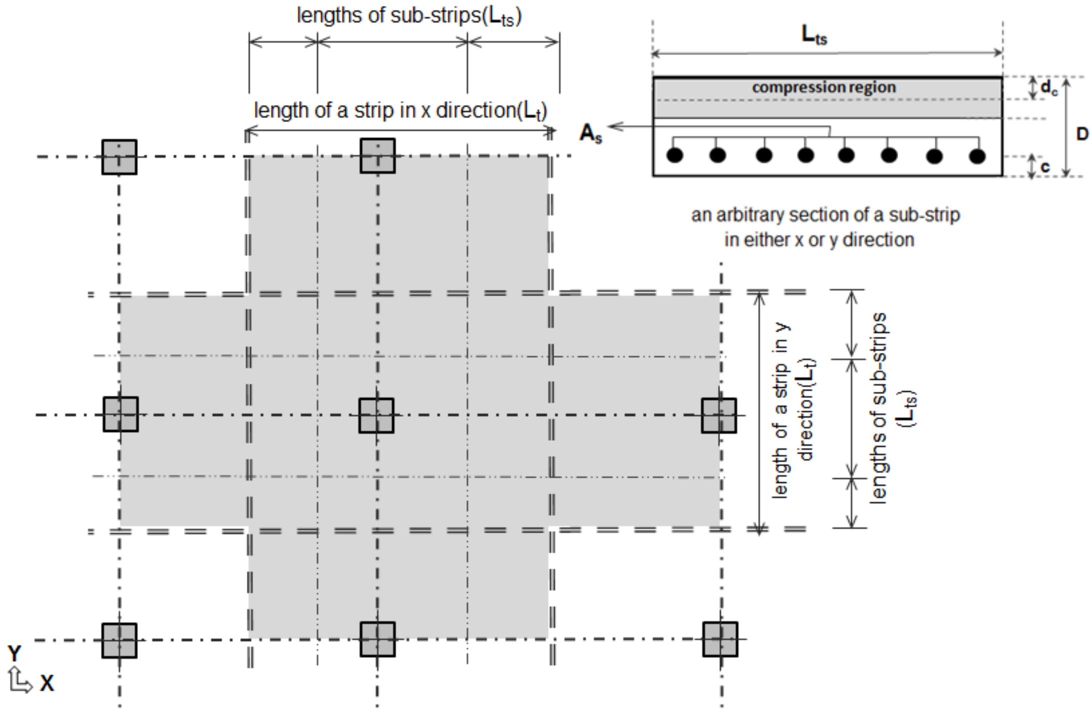

Figure 1. Equivalent frame strips for flat plates

In the equivalent frame method, the only parameters which are required to design a flat slab are the distributed bending moments in the x and y directions over the floor. Therefore, if a function identifies the relation between the distributed bending moment and the cost for an arbitrary section, a new cost function with new variables will be achieved. Consider Eq. (3) as a potential alternative cost function to Eq. (1) in an arbitrary slab cross-section.

$$\begin{cases} C^{(s)} = c_m M_u \\ C = \sum_1^{N_s} C^{(s)} \end{cases} \tag{3}$$

where M_u is the bending moment capacity of an arbitrary section of the flat slab in the x or y direction and $C^{(s)}$ and N_s are the cost of each section and the total number of control sections in the floor respectively. If an appropriate c_m was found in such a way that Eq. (3) represented the cost of the sections, the design variables would shift from A_c, A_s and P_f to M_u. In fact, due to the relationship between the capacity factors of the section and structural analysis outputs in design standards, the aim of using cost functions such as Eq. (3) is to use structural analysis outputs instead of structural design factors.

The advantages of using parameter M_u in place of A_c, A_s and P_f are: firstly, M_u can be easily obtained from structural analysis, and in an iterative procedure, re-analyzing a structure is considerably less time-consuming and more precise than re-designing the structure. Particularly, in case of flat slabs and using the equivalent frame method bending moments in each section can be easily obtained by simplified formulations and the design standards tables. If Eq. (1) is used as an objective function for finding the optimum cost of a large structure or in multi-objective or multi-variable optimisation of a structure, the optimisation tool needs to deal with both structural analysis and structural design in each step in order to move towards an optimum solution. Therefore, the aim is to introduce a cost function and consequently a method that only deals with structural analysis parameters in the optimisation process in order to find the optimal layout of a flat slab floor system.

FORMULATION OF THE COST FUNCTION

Based on the equivalent frame method, each strip is divided into a number of sub-strips in order to receive the distributed moments, and each sub-strip is considered an equivalent beam. As shown in Figure 1, the dimensions of the section of each sub-strip are L_{ts} and D, and the cross-sectional area of reinforcement for such a section is A_s. The capacity or the ultimate strengths of the section in flexure in either direction is M_u, which can be obtained from Eq. (4).

$$\begin{cases} M_u \cong A_s\, f_y\, (D - c - d_c) \\ d_c = 0.5\gamma k_u (D - c) \end{cases} \tag{4}$$

where f_y is the characteristic strength of the longitudinal reinforcing steel, and d_c is the distance from the extreme compression fibre of the concrete to the compressive force in either direction. The coefficients γ and k_u are calculated based on the characteristic strength of the concrete and reinforcing steel, and c is the cover to the reinforcement steel (Warner et al., 1998).

In order to shift from Eq. (1) to Eq. (3) and come up with the set of $\{c_i\}$, first, the reciprocal relationship between the variables of Eq. (1) and the variables of Eq. (3) need to be identified. That is, it needs to be determined how variations of A_c, A_s and P_f affect M_u and vice versa and how increasing or decreasing the amount of each cross-sectional feature influences the section strength capacity, and how one should change the cross-sectional parameters to vary section capacity and consequently the floor capacity. In layout optimisation of slab floors, the term of formwork cost can be removed from the calculation process, because the total area of slabs is constant, and floor layout and span lengths has no effect on the final amount of formwork. Therefore, the terms P_f and c_f can be removed from Eq. (1). Now, given the unit costs c_c and c_s as relative unit prices for area of a section, the cost function can be defined using Eq. (1) for each section. If any of the cross-sectional parameters A_c or A_{sl} changes, the cost varies as follows

$$\Delta C = c_c\, \Delta A_c + c_s\, \Delta A_s \tag{5}$$

On the other hand, using Eq. (3) variations in section capacities would change the cost function as follows

$$\Delta C = c_m\, \Delta M_u \tag{6}$$

Eqs. (5) and (6) show the contribution of each factor to cost changes and sensitivity of the cost to each term. For example, changing a unit of A_c, causes a change of c_c units in cost. Therefore, if the effect of variations of A_c and A_s on variations of M_u are determined, the contribution of the section capacity to cost changes, that is the coefficient c_m, can be determined.

There are two variables in Eq. (5); steel reinforcement area A_s and section depth D. If A_s varies:

$$\frac{\Delta M_u}{\Delta A_s} \cong f_y\, (D - c - d_c) \rightarrow \Delta A_s \cong [f_y\, (D - c - d_c)]^{-1}\, \Delta M_u = K_1\, \Delta M_u \tag{7}$$

If D varies the section area A_c varies as follows:

$$\Delta A_c \cong L_{ts}\, \Delta D \tag{8}$$

The variation of the section capacity can be written as

$$\frac{\Delta M_u}{\Delta A_c} = \frac{\Delta M_u}{L_{ts}\,\Delta D} \cong \frac{f_y A_s}{L_{ts}}(1 - 0.5\gamma k_u) \;\rightarrow\; \Delta A_c \;\cong\; [\frac{f_y A_s}{L_{ts}}(1 - 0.5\gamma k_u)]^{-1}\,\Delta M_u = K_2\,\Delta M_u \quad (9)$$

Multiplying both sides of Eq. (7) by c_s and Eq. (9) by c_c, and adding them up will result in:

$$c_c\,\Delta A_c + c_s\,\Delta A_s = \;(c_s K_1 + c_c K_2)\,\Delta M_u \quad (10)$$

Comparing Eq. (10) with Eq. (6) results in:

$$c_m = \;c_s K_1 + c_c K_2 \quad (11)$$

The coefficient c_m determines how the parameter M_u contributes to the cost function. Now, in order to re-analyze the flat slab to achieve the optimum criteria, one can use Eq. (3) in lieu of Eq. (1). For this purpose, and using Eq. (3), the cost will be the sum of cost functions of all selected sections in the structure.

$$C_t = \sum_{i=1}^{NS} C_i = \sum_{i=1}^{N_S} c_{mi}\,M_{ui} = \sum_{i=1}^{N_S}(c_s K_{1i} + c_c K_{2i})M_{ui} \quad (12)$$

where C_t is the total cost of the flat slab floor and N_S is the number of selected sections to control the cost. In the equivalent frame method, if the number of spans in *x* and *y* directions are respectively N_X *and* N_Y, the number of control sections for the multi-span flat slab is $(3*N_Y+1)(3*N_X)$ for the *x* direction and $(3*N_Y+1)(3*N_X)$ for the *y* direction. Therefore the total number of control sections for the entire floor is

$$N_S = (3*N_Y+1)(3*N_X) + (3*N_Y+1)(3*N_X) \quad (13)$$

Based on AS-3600 (2009), the deflection calculations can be avoided, if the effective depth is in accordance with the rules for allowable span/depth ratio. So, based on this rule, the depth constraints on the whole area and all sections of the floor under a load case may be written as:

$$\frac{L_n}{D} \leq 55[\frac{L_n/L'_n}{wK_p}]^{1/3} \quad (14)$$

in which L_n and L'_n are the longer and shorter clear spans of slabs, *w* is service load including self weight and K_p is a coefficient to distinguish between interior and exterior panels. Other constraints for durability, fire resistance, minimum cover and minimum flexural strength, can be easily added to the problem as well, based on the relevant design standards.

It should be noted that in flat slabs, when designing drop panels, the required flexural and shear strengths of the regions around the columns are determined according to the total unbalanced moment to be transmitted to the column and the shear strength of the total perimeter around columns respectively. Therefore, such requirements need to be considered as the explicit constraints in the optimisation problem. Moreover, the size of drop panels are considerably smaller compared to the entire floor and their dimensions do not change significantly by variations of the spans lengths. As a result, the effects of changes in the drop panels due to changes in the spans lengths on the optimisation procedure are considered negligible.

Now, consider a multi-span RC flat slab with N_x and N_y spans and total lengths of L_x and L_y in the x and y directions under the arbitrary loading system $f(x)$. The aim is to re-design the floor to determine the optimum span lengths in each direction in order to minimize the cost. The final cost will be a function of two sets of variables. According to Eq. (13), the total cost is a function of the sections' action effects under a loading system, which in turn are functions of the span lengths based on Eqs. (2). If NSP is the total number of spans in x and y directions the general formulation of the problem is:

$$\begin{cases} \min\limits_{l_1, l_2, \ldots, l_{NSP}} \quad Cost(l_1, l_2, \ldots, l_{NSP}) = \sum_{i=1}^{NS} (c_s K_{1i} + c_c K_{2i}) M_{ui} \qquad for\ i = \{1,2, \ldots NS\} \\ \text{-------------------} \\ s.t \begin{cases} \emptyset_f \{M_{ui}\} \geq \{M_{oi}\} \\ \frac{L_{ni}}{D} \leq 55 \left[\frac{\frac{L_n}{L'_n}}{wK_p} \right]^{\frac{1}{3}} \\ (V_u^{(drop)} \leq V^{*(drop)}) \wedge (M_u^{(drop)} \leq M^{*(drop)}) : \quad For\ all\ drops \\ \{L_{min}\} \leq \{L_i\} \leq \{L_{max}\} \\ other\ constraints\ based\ on\ the\ design\ standard \end{cases} \end{cases} \tag{15}$$

Since the strips' lengths are functions of span lengths, making use of Eqs. (2), and substituting parameters K_{1i} and K_{2i} from Eqs. (7) and (9), the variables of the above optimisation problem will be the spans lengths and the slab depth. Although in theory the size of RC members and their dimensions in building structures are continuous, in the design process, we mainly deal with the dimensions as discrete sizes. The dimensions of concrete sections or span lengths are usually varied by a certain size, e.g. 25 mm or 50 mm a step, which makes the section dimensions discrete. Therefore, one can define the layout optimisation problem of multi-span floors as a discrete optimisation problem. There are several ways to deal with the above cost optimisation problem.

A Comparative Numerical Example

A flat slab floor system with the total length and width of $L_x = L_y = 37.5$ m and the number of spans of $N_x = N_y = 7$ in either direction is considered. Dead load and live load are 1.5 kPa and 5 kPa respectively. The average unit price for concrete, reinforcement and formwork are respectively assumed to be 53.5 units/m^3, 3120 units/m^3 and 18.5 units/m^2. Other design parameters are $f_y = 460$ MPa, $f'_c = 35$ MPa, and $c = 25$ mm. Considering Eq. (15) and using the ACO algorithm and meeting the requirements stated in Clause 7.4.1 of AS3600 (2009) as the constraints of the problem and after 120 iterations, and at CPU time of 16.09 seconds, the following results were obtained. The optimum lengths are $L_{x1} = L_{x7} = 5050$ mm, $L_{x2} = L_{x6} = 5100$ mm, $L_{x3} = L_{x5} = 5150$ mm and $L_{x4} = 6900$ mm. Due to the symmetry of the plan, the lengths of spans in the y direction are the same as the ones in the x direction. The obtained spans result in a total cost of 46950 units (11.1% cost saving). Figure 2 shows a typical convergence history using the proposed ACO algorithm for the example.

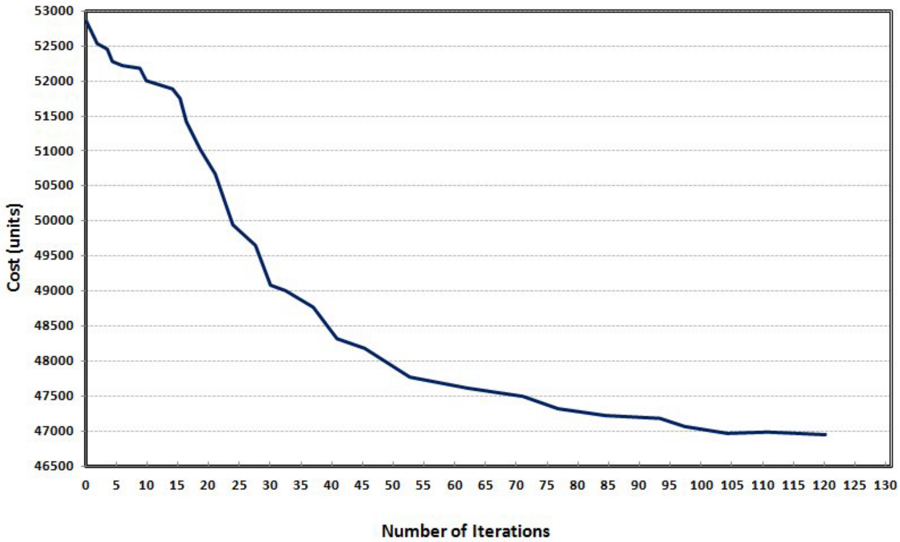

Figure 2. A typical convergence history for the ACO algorithm

CONCLUSIONS

The main objective of this study is to propose an alternative model to traditional objective functions, used for cost optimisation of RC slabs, which can be easily used for layout optimisation of multi-span flat slabs. In the proposed formulation, unlike the commonly used cost optimisation methods, there is no need to deal with the design variables like cross-sectional parameters. The variables of such a formulation are the action effects that can be easily obtained from a structural analysis procedure instead of the cross-sectional variables that are typically the results of a design procedure. The cost function proposed in this study simplifies the process of cost optimisation of multi-span flat slabs and is applicable to multi-variable optimisation of such structures.

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