Statistical Multi-Stage Artificial Neural Network for Damage Detection with Consideration of Unmeasured Mode Shape Data

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ABSTRACT
The problem of the limited number of sensors and the existence of uncertainties due to measurement and modeling errors is still the leading setback in vibration-based damage detection technology. The limited numbers of sensor may result into the structural vibration properties not being sensitive to small damage in a large element, while the existence of uncertainties may lead to a false and unreliable result in damage detection. Due to the ability of ANN and statistical method to handle nonlinear relationship and the existence of uncertainties respectively, this study demonstrates a new method which combines a multi-stage ANN model and statistical method to detect damage based from the limited number of sensors with consideration of uncertainties. The first stage of ANN is used to predicting the unmeasured mode shapes from the limited measured modal data. The second stage ANN is devoted to predict the damage location and severity using the complete modal data from the first stage ANN. To incorporate the uncertainties in modal data, Gaussian noise is applied to the input variables and the probability of damage existence is calculated using Rosenblueth's point estimate method. The feasibility of the proposed method is demonstrated using an analytical model of a continuous two-span reinforced concrete slab.

KEYWORDS
Limited number of sensors; Artificial neural network; Uncertainties; Damage.

INTRODUCTION

Civil structures are vulnerable asset to human and therefore it is extremely beneficial to maintain them from fast deterioration at an alarming rate. Traditionally, structural engineers conduct checks on structures based on physical inspections for any defects after the occurrence of a disaster. As the frequency of the structural failures is getting more prominent, hence the awareness of the importance of the structural health monitoring has arisen. Many structural engineers have started to consider the advancement of the developed technologies to help in increasing civil structure lives and reducing the cost of structure maintenance. The trend has been changing from utilising the localised methods such as radiographs to global methods using dynamic properties. Changes in dynamic properties depict the changes in structural properties, which indicate the present of damage existence. In accessing structure using dynamic properties, reliable information comprises of a sufficient number of measurements points is necessary to ensure an accurate result. Somehow it is rather difficult to use a large number of sensors due to cost constrains and accessibility. Hence the need to establish a technique to detect structural damage based on a limited number of sensors is essential.

Uncertainties in modeling and measurement will cause the damage identification to be less precise. Recent studies that have been conducted to incorporate the effect of uncertainty in damage identification process are as follow. Lew (2008) employed the feedback control to reduce the effect of model uncertainty in damage identification. The proposed method used a small number of sensors in the correlation approach to investigate the effect of model uncertainty, boundary condition variation and natural frequency changes. Although the proposed method is capable to reduce the effect of boundary condition uncertainty, it has only applied to a simple cantilever Euler's beam. Wang *et al.* (2010) proposed the inner product vector (IPV) as the damage detection algorithm which uses cross correlation functions between response measurements to eliminate some particular measurement noise. Damage is identified by sudden changes in IPV of the intact and damaged structure. The authors illustrated the proposed method with three damage detection experiments. Nevertheless, the damage extent is not quantifiable, and moreover the sensor locations should be placed near to the actual damage to obtain a satisfactory result. This requires prior knowledge in term of damage location to be known in advance.

Zhang (2007) conducted statistical damage identification in bridges to tackle uncertainties issue in real bridges. The author employed a 4-steps process involving data segmentation, data normalisation, damage feature extraction and statistical interpretation. The ambient vibration responses are used as input data. The practicality of the proposed procedure is demonstrated in a 3 spans numerical bridge model. Bakhary *et al.* (2010) studied on the substructuring method in damage detection. In this study, each substructure is assessed independently with using statistical ANN to consider the uncertainties in the ANN input data. Xu *et al.* (2011) employed a stochastic method to compute the probability density function (PDF) of the structural stiffness parameter randomly before identify damage locations and their corresponding severities using probability of damage existence and the PDFs of damage severity indices. The proposed method is demonstrated using a 3-storey shear building structure. Goller and Schueller (2011) investigated the feasibility of Bayesian statistical concept in model updating procedures considering model uncertainties in a numerical beam example. Although previous studies are able to demonstrate the applicability of the statistical method to consider uncertainties, the issue related to the limited number of sensors is still unsolved.

In this study, an approach using a multi-stage artificial neural network (ANN) is proposed for damage detection in considering the limited number of sensors in damage detection. Moreover, the statistical method employing Rosenbleuth's point estimate method is also applied to consider the existence of uncertainties due to measurement and modelling errors. Two single ANN systems are used to form the multi-stage ANN model. The model is consists of two stages; first is to predict the mode shape value at the locations without the sensors being placed, while the second stage is for damage identification. A numerical continuous two-span reinforced concrete slab model is employed as an example in this study. A simulated damage case is then used to test the feasibility of the proposed method.

METHODOLOGY

This study employed a multi-stage stage ANN where ANN model in the first stage (ANN1) is used to predict the mode shape values at the unmeasured points while the ANN model in the second stage (ANN2) is used to determine the damage severities and locations. The ANN1 model is trained to relate between the frequencies and mode shape values with the unmeasured mode shape values, while ANN2 employs the predicted mode shapes values together with the measured frequencies and mode shape to determine the damage location and severity. To consider the uncertainties; Gaussian noise is applied in the training data of the ANN models with zero mean and one standard deviation. The final outputs from the multi-stage ANN are used to estimate the probability of damage existence (PDE) using Rosenblueth's point estimate method (PEM).

ANN Architecture

As mentioned earlier, the multi-stage ANN is fractioned into ANN1 and ANN2 as demonstrated in Figure 1. In ANN1, the inputs are the first three mode shapes (ϕ) and frequencies (ψ) measured at the selected limited sensor points while the outputs are the mode shape values at the unmeasured nodal points (ψ_u) along the centerline of the structure. For training purpose, 3000 training data are used in the ANN model. These training data consist of various damage cases that are generated from the finite element model. The damage cases are simulated by reducing the Young's modulus (E) in the selected segments. The damage cases are varied between the ranges of 0.3xE and 1.7xE. The training data are divided into 70%, 20% and 10% respectively for training, validation and testing purposes.

In ANN2, the inputs are a combination of all measured mode shape values, frequencies and unmeasured mode shape values from ANN1. The outputs of ANN2 are the damage index (DI) which will be used to compute through the statistical method for the PDE values. The noise is assumed to be independent of each other. In both ANN1 and ANN2, the numbers of hidden neurons are determined by trial and error.

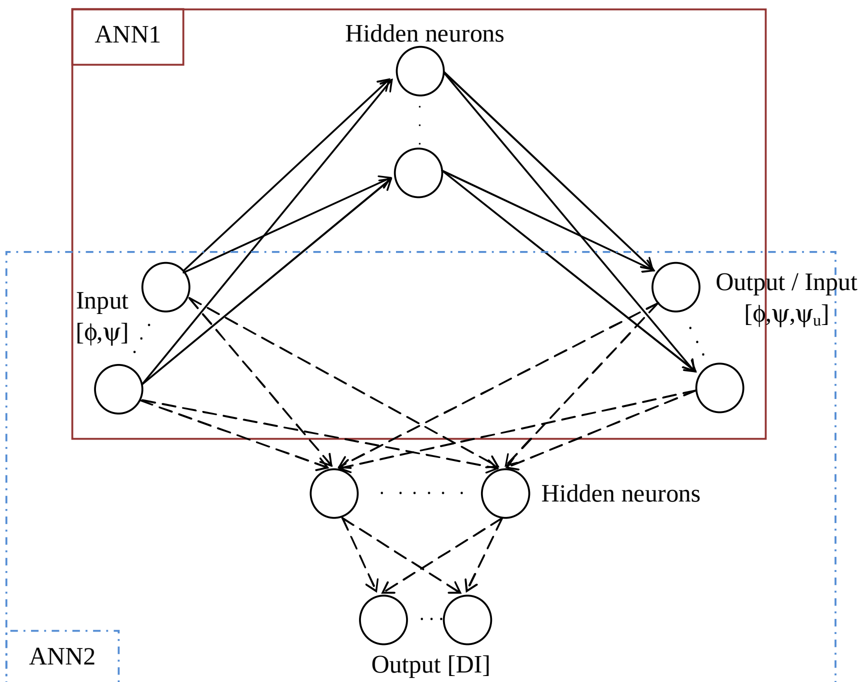

Figure 1. Developed multi-stage ANN Model.

Probabilistic multi-stage ANN

To consider noise, the inputs in the ANN1 will be smeared with random noise to represent uncertainty using the approach in PEM. In PEM, the Gaussian noise will be added to the ANN1 inputs by the mean of plus and minus one standard deviation to both mode shapes and frequencies in the training data in the ANN models. As there are two parameters as random variables involved, there will be a total of four functions rendered, viz.:

$$\alpha_{j\,++} : [\phi_i + \sigma_\phi,\ \psi_i + \sigma_\psi] \quad [1]$$

$$\alpha_{j\,--} : [\phi_i - \sigma_\phi,\ \psi_i - \sigma_\psi] \quad [2]$$

$$\alpha_{j\,-+} : [\phi_i - \sigma_\phi,\ \psi_i + \sigma_\psi] \quad [3]$$

$$\alpha_{j\,+-} : [\phi_i + \sigma_\phi,\ \psi_i - \sigma_\psi] \quad [4]$$

$\sigma_{(\phi \text{ or } \psi)}$ is the standard deviation that is applied in the training data (mode shape or frequency), ϕ_i and ψ_i are the measured mode shape and frequency values at *i*th case respectively, and (α_{j++}, α_{j--}, α_{j-+}, α_{j+-}) are the target outputs of the trained ANN1. There will be four iterations to be completed in the training of the ANN model with different configuration of function as expressed above. In each of the iteration, the trained ANN model is tested with four configurations of function in the same manner. Subsequently, the mean and standard deviation of the values of from the four iterations models will be then evaluated using the following equations:

$$\text{Mean, } \mu(\alpha) = \frac{1}{16}\{\alpha_j{}^1{}_{++} + \alpha_j{}^1{}_{--} + \alpha_j{}^1{}_{-+} + \alpha_j{}^1{}_{+-} + \ldots\ldots\ldots + \alpha_j{}^4{}_{++} + \alpha_j{}^4{}_{--} + \alpha_j{}^4{}_{-+} + \alpha_j{}^4{}_{+-}\} \quad [5]$$

$$\text{Standard deviation, } \delta(\alpha) = \{\mu(\alpha^2) - (\mu(\alpha))^2\}^{1/2} \quad [6]$$

The basic principle of PEM can be accessed in Rosenblueth (1975) and a more detail explanation on PEM can be referred to Bakhary *et al.* (2010). Once the $\mu(\alpha)$ and $\delta(\alpha)$ are computed, then the PDE values can be calculated from the statistical distributions of the damage index in each segment of the undamaged and damaged models. The probability density functions for undamaged and damaged models are plotted. A lower bound of the undamaged model is set at $\{\mu(\alpha) - 1.645\delta(\alpha)\}$, with 95% confidence level. The intersection area between the two models will give the approximation of the relationship between two models. The larger is the intersection area, the more related of the two models, resulting in non- or lower damage existence and vice versa. PDE is calculated by determining the probability of the damaged model in the 95% confidence healthy interval. Thus, PDE is constituted of the shaded area as exemplified in Figure 2.

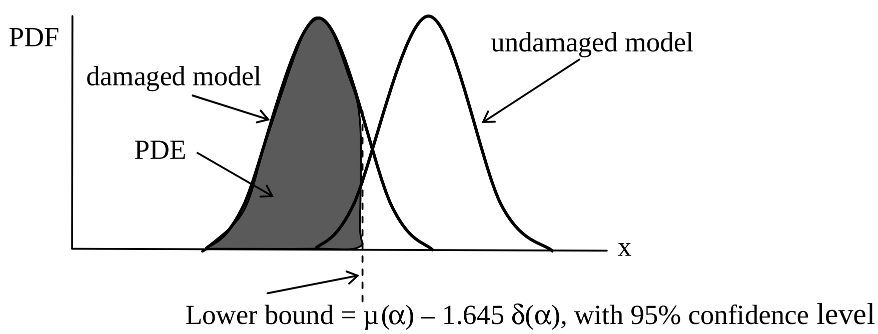

Figure 2. Probability density functions plots to evaluate PDE values.

Numerical Example

The numerical example employed in this study is a continuous two-span reinforced concrete slab model to demonstrate the proposed multi-stage ANN approach for damage detection. The slab is modeled using Structural Dynamics Tools (SDT) which runs on Matlab platform. The slab dimensions are 6.4 m in length, 0.8 m in width and 0.1 m thickness. The slab is simply supported at 0.2 m, 3.2 m and 6.2 m from the left end, Young modulus is 3.3×10^{10} N/m^2, the density is 2.45×10^3 kg/m^3 and the poisson ratio is 0.2. The slab is modeled using shell elements with 165 nodes as shown in Figure 3(a), and supports are assumed to be simply supported along nodes 6 to 10, 81 to 85 and 156 to 160. The modal analysis is conducted using finite element analysis, and the mode shape values are measured at every 0.2 m of the centerline of the slab model along the span. For the purpose of damage detection, the slab is divided into 32 segments whereby each segment is equally distributed vertically of area size 0.2 m x 0.8 m as shown in Figure 3(b).

The tests are carried out once the ANN models are fully trained with different percentage of uncertainties in both mode shapes and frequencies values. In the first stage, the developed trained

ANN models are utilised to predict the mode shape values at the unmeasured points. In this study, the ANN1 inputs comprise of the first three mode shapes and frequencies measured at the selected limited sensor points at the 16 nodal points as indicated in Figure 3(a). There are a total of 33 points along the slab span in a complete set. Nevertheless there are 2 points that are located at the supports and thus are eliminated in the model. Consequently, only 15 unmeasured measurement points will be anticipated in ANN1 models. The details of the measured and unmeasured nodal points in the finite element model are as tabulated in Table 1. ANN2 models are then being employed to incorporate both inputs and outputs from ANN1 as inputs into the second stage model to detect the damage index. PDE is calculated from the outputs of the ANN2. In this study, the damage is simulated with a reduction of Young's modulus in segment no. 4, 5, 12, 20 and 26. The reduction is designated at 40%, 30%, 20%, 30% and 40% from the Young's modulus respectively.

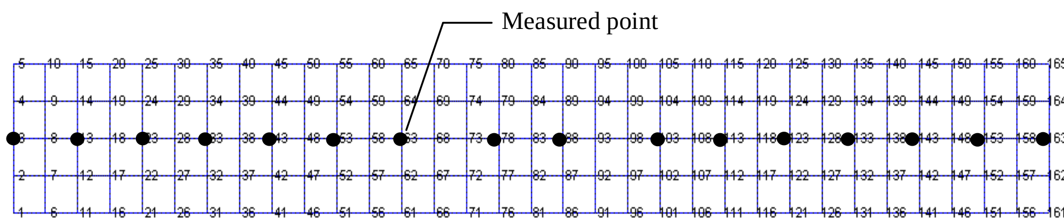

Figure 3(a). Finite element model of a two-span concrete slab.

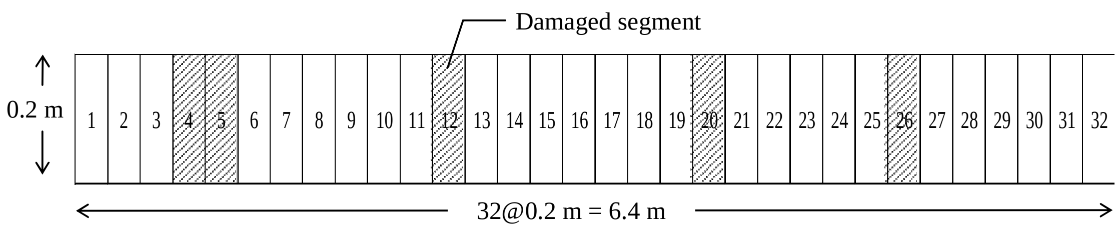

Figure 3(b). Slab segments on finite element model.

Table 1. List of measured and unmeasured nodes.

Measured nodal point	Unmeasured nodal point
3, 13, 23, 33,43, 53, 63, 78, 88, 103, 113, 123, 133, 143, 153 and 163	8, 13, 18, 28, 33, 38, 48, 53, 58, 68, 73, 78, 83, 88, 93, 98, 108, 113, 118, 128, 133, 138, 148, 153 and 158

RESULTS AND DISCUSSION

A damage detection method with consideration of a limited number of sensors is presented. A multi-stage ANN is proposed for locating the damage location and quantifying the damage severity. The feasibility study of the efficiency of ANN1 to predict the unmeasured mode shape values has been attained in Goh *et al.* (2011). ANN1 is capable to predict all the unmeasured measurement points even with a limited number of measurement points. Comparison study has been conducted with the cubic spline interpolation method which has been employed by many researchers to conduct mode shape expansion. The deterministic results have also been discussed in the previous paper. There are some minor negative and positive false detection on the left and right of the damaged segments in the slab as depicted in Figure 4. The false detection is due to the effect of progressive error in the ANN. Thus, this paper focuses on the potential of using the multi-stage ANN for damage identification to overcome the false detection in the deterministic approach. The proposed method employs only a limited number of input data with the consideration of uncertainties incorporating with the Rosenblueth's point estimate method.

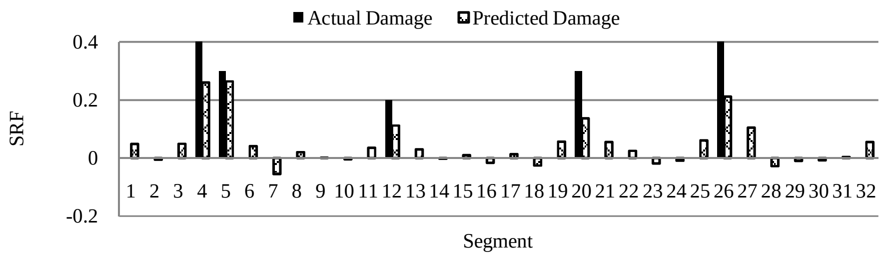

Figure 4. Results of damage identification in the deterministic model.

Relationship of PDE values to the damage severity

This section is to investigate the sensitivity of the proposed multi-stage ANN to different levels of damage severities. For this purpose, the ANN models are developed with uncertainty level of 10% for mode shapes and 1% for frequencies in the training stage. After the ANN model is trained, it is applied to test with the simulated damage at the designated segments as mentioned earlier with damage severity of -5%, -10% to -60% with every increment of -10% in *E*. Thus, the simulated damage is equivalent to 0.95*E* to 0.4*E* respectively. The reduction of *E* rendered in each of the damaged segment is assumed to be unvarying in each damage level. PDE values are calculated and presented in Table 2. The PDE values of the damaged segments are highlighted in the table.

It is observed that the PDE values obtained at damaged segments are higher compared to the undamaged segments in the slab. This evidenced that the developed ANN models are capable of detecting the locations of the damages. It is also observed that the highest PDE values occurred in the case of -60% damage level compared to the rest. The results indicated that when the damage level is higher, the damage can be detected with higher confidence. When the damage occurrence is small in a structure especially in a large structure, the changes of modal parameters are not significant. This explains why the PDE values acquired in the lower damage level is smaller. The relationship between the calculated PDE values and the damage severity is as presented in Figure 5. From the figure, the PDE values increase with the increase of the damage severity. This indicates that the damage severity has influence on the damage detection in the proposed method.

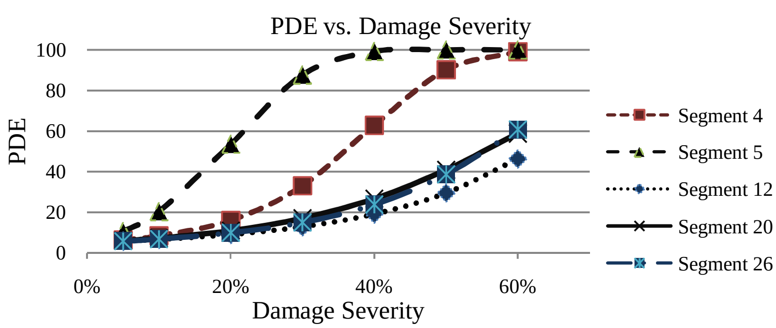

Figure 5. PDE vs. Damage Severity.

Relationship of PDE values to different levels in training data

This section presents the effect of different uncertainty levels in the training data to the PDE values. The ANN models are trained using different percentages of uncertainty level induced in the input data of ANN1. The percentage of uncertainty applied in the study is 5% mode shape and 0.5% frequency, 10% mode shape and 1% frequency, 15% mode shape and 2% frequency, and 20% mode shape and 5% frequency. Although the uncertainty levels differ in the training data, the testing data consists of identical uncertainty level of 5% uncertainty in mode shape and 0.5% uncertainty in frequency. The trained ANN models are also tested with the noise-free testing data. The result obtained is tabled in Table 3.

Table 2. PDE values vs. damage severities.

Training		Testing			Segment																															
ϕ	ψ	ϕ	ψ	E	1	2	3	**4**	**5**	6	7	8	9	10	11	**12**	13	14	15	16	17	18	19	**20**	21	22	23	24	25	**26**	27	28	29	30	31	32
10%	1%	10%	1%	-5%	5.0	4.9	4.8	**6.4**	**10.6**	5.1	4.7	5.1	4.9	4.7	5.4	**5.7**	5.8	4.9	4.7	5.0	4.9	5.3	5.1	**6.0**	5.0	4.8	4.9	5.0	5.6	**5.8**	5.4	4.7	4.4	5.3	5.2	4.6
				-10%	4.9	4.7	4.6	**8.4**	**20.3**	5.1	4.3	5.2	4.7	4.5	5.8	**6.6**	6.8	4.8	4.5	5.0	4.8	5.6	5.2	**7.3**	5.1	4.6	4.8	4.9	6.2	**6.9**	5.8	4.5	3.9	5.6	5.3	4.2
				-20%	4.9	4.5	4.0	**16.1**	**53.6**	5.1	3.6	5.5	4.5	4.0	6.6	**9.0**	9.5	4.4	3.8	5.0	4.5	6.4	5.5	**11.0**	5.3	4.3	4.5	4.9	8.1	**9.9**	7.0	4.1	2.9	6.4	5.7	3.5
				-30%	4.8	4.3	3.2	**33.1**	**87.6**	5.1	3.0	5.8	4.2	3.4	7.4	**12.8**	13.3	4.1	3.1	4.9	4.2	7.2	5.9	**17.0**	5.7	3.8	4.3	5.0	10.9	**15.0**	8.8	3.8	2.2	7.6	6.2	2.9
				-40%	4.9	4.1	2.3	**62.8**	**99.3**	5.0	2.4	6.1	4.0	2.8	8.2	**19.1**	18.8	3.6	2.4	4.7	3.9	8.2	6.6	**26.7**	6.3	3.4	4.2	5.3	15.8	**23.9**	11.4	3.9	1.6	9.4	6.6	2.4
				-50%	5.0	3.9	1.3	**90.2**	**100.0**	4.9	2.1	6.6	3.9	2.0	9.0	**29.5**	26.5	3.1	1.5	4.5	3.6	9.0	7.8	**40.9**	7.4	2.8	4.1	5.9	24.9	**38.7**	15.5	4.4	1.1	12.4	6.9	2.2
				-60%	5.4	3.5	0.5	**99.1**	**100.0**	4.8	2.0	7.7	4.3	1.1	9.7	**46.3**	36.5	2.6	0.8	4.4	3.3	9.4	9.8	**58.8**	9.1	2.2	4.1	6.9	43.3	**60.7**	21.6	6.1	0.6	17.6	6.9	2.3

Table 3. PDE values vs. noise level in ANN training data.

Training		Testing		Segment																															
ϕ	ψ	ϕ	ψ	1	2	3	**4**	**5**	6	7	8	9	10	11	**12**	13	14	15	16	17	18	19	**20**	21	22	23	24	25	**26**	27	28	29	30	31	32
5%	0.5%	0	0	0.2	3.3	49.2	**100.0**	**100.0**	14.1	1.0	14.9	13.0	7.4	59.3	**65.5**	34.6	2.7	0.8	14.3	15.6	8.8	13.8	**100.0**	42.0	26.4	0.8	0.7	99.7	**100.0**	97.8	3.5	0.0	45.3	26.8	0.0
		5%	0.5%	5.8	4.9	10.4	**100.0**	**100.0**	7.4	1.1	7.5	7.0	6.2	26.8	**18.1**	10.5	4.6	3.5	7.7	5.6	3.8	6.8	**62.5**	14.0	11.0	4.3	1.5	61.3	**81.7**	36.8	7.0	1.4	10.9	6.6	2.4
10%	1%	0	0	6.2	6.4	3.5	**97.7**	**100.0**	5.5	5.5	6.4	2.9	7.6	6.7	**14.3**	12.7	3.1	0.9	6.9	4.8	5.4	12.1	**76.9**	8.1	4.0	3.7	1.9	78.6	**55.8**	28.5	6.0	0.0	12.9	29.2	1.0
		5%	0.5%	7.5	6.8	4.2	**99.9**	**100.0**	3.4	6.5	7.2	4.0	7.5	5.6	**26.3**	16.0	3.3	2.2	10.8	4.7	5.2	11.2	**55.4**	10.0	3.3	3.1	1.9	57.3	**70.5**	57.7	5.7	0.1	15.8	18.8	1.1
15%	2%	0	0	8.5	5.0	13.9	**49.7**	**74.0**	11.4	3.6	35.3	0.6	3.6	28.0	**18.2**	16.5	4.1	4.3	3.9	3.5	6.2	8.1	**60.4**	10.9	1.1	5.2	6.0	9.8	**18.2**	22.3	10.9	1.0	10.2	6.1	5.5
		5%	0.5%	8.1	6.2	17.3	**55.9**	**84.2**	12.4	3.3	16.4	5.7	3.2	16.0	**22.9**	15.8	4.6	4.5	4.7	4.3	7.5	8.4	**48.7**	10.8	4.2	5.3	5.8	10.7	**20.5**	23.4	9.8	1.5	9.2	6.0	4.8
20%	5%	0	0	4.8	3.4	8.5	**12.8**	**61.9**	24.3	5.4	6.0	5.6	4.7	6.5	**6.9**	5.3	5.1	5.0	6.0	3.6	5.5	8.0	**7.8**	4.2	8.8	4.6	3.0	7.8	**9.1**	11.7	3.7	3.4	6.5	7.4	4.2
		5%	0.5%	4.5	3.8	11.0	**26.9**	**71.4**	5.5	4.4	6.3	5.8	4.1	7.1	**8.0**	5.6	5.0	4.9	5.7	3.7	5.7	9.6	**9.5**	6.5	10.5	2.7	3.6	9.7	**11.2**	15.9	4.7	3.3	6.4	5.8	4.1

Table 4. PDE values vs. other noise level in ANN training data.

Training		Testing		Segment																															
ϕ	ψ	ϕ	ψ	1	2	3	**4**	**5**	6	7	8	9	10	11	**12**	13	14	15	16	17	18	19	**20**	21	22	23	24	25	**26**	27	28	29	30	31	32
5%	0.5%	10%	1%	5.7	5.2	7.3	**98.9**	**97.7**	5.6	2.1	6.5	5.0	5.9	12.1	**10.5**	6.2	4.8	3.7	6.5	5.3	4.9	5.6	**31.4**	8.1	8.7	4.0	2.4	21.3	**38.0**	19.9	6.5	3.0	8.5	4.2	3.2
10%	1%	10%	1%	6.4	4.9	4.6	**50.6**	**86.6**	4.6	4.5	7.6	4.7	5.0	6.0	**8.7**	8.2	4.1	3.2	5.6	4.9	5.1	6.3	**8.3**	6.2	3.2	5.3	4.1	12.6	**14.7**	8.5	5.9	1.8	5.2	7.0	1.1
15%	2%	10%	1%	6.5	4.9	11.3	**56.2**	**85.2**	9.2	3.3	12.3	4.6	2.9	14.3	**21.2**	13.2	5.0	4.7	4.7	4.8	8.5	8.8	**25.0**	10.5	4.9	5.6	5.7	12.1	**22.4**	24.4	7.4	1.9	8.7	6.3	3.8
20%	5%	10%	1%	6.9	5.7	7.1	**9.6**	**28.5**	7.3	4.6	6.2	6.0	4.8	6.2	**7.3**	5.3	5.6	3.9	5.4	3.0	5.9	6.1	**8.0**	4.2	6.7	5.7	3.2	6.2	**6.5**	6.9	3.7	4.5	7.7	5.6	4.1

Training		Testing		Segment																															
ϕ	ψ	ϕ	ψ	1	2	3	**4**	**5**	6	7	8	9	10	11	**12**	13	14	15	16	17	18	19	**20**	21	22	23	24	25	**26**	27	28	29	30	31	32
5%	0.5%	15%	2%	6.3	5.7	6.6	**70.9**	**64.6**	5.3	4.1	6.7	5.1	6.0	9.4	**8.1**	6.6	5.1	3.9	5.8	5.7	5.3	5.3	**18.0**	6.6	7.9	5.7	3.8	12.4	**23.1**	14.2	5.7	4.0	7.3	4.1	3.9
10%	1%	15%	2%	6.7	4.7	3.0	**62.4**	**92.9**	4.0	6.9	6.9	4.7	4.9	4.5	**10.9**	6.7	4.5	3.6	5.7	4.7	6.4	5.8	**7.8**	6.2	2.6	5.3	4.8	11.7	**12.1**	8.5	5.8	1.9	5.7	5.4	1.7
15%	2%	15%	2%	6.4	5.2	4.5	**25.2**	**27.3**	4.8	5.2	6.1	3.3	4.7	7.0	**9.4**	7.2	5.7	4.0	3.5	4.5	5.2	11.5	**15.2**	15.5	4.6	5.5	6.0	8.9	**20.6**	9.8	5.6	3.5	5.5	2.5	7.1
20%	5%	15%	2%	7.3	6.0	6.9	**16.7**	**49.5**	6.6	4.0	5.9	5.7	5.1	6.7	**8.0**	5.8	5.6	4.2	6.0	3.9	6.4	6.3	**8.5**	7.3	5.9	4.1	2.9	6.7	**7.8**	9.0	3.8	5.3	7.2	5.4	4.5

Training		Testing		Segment																															
ϕ	ψ	ϕ	ψ	1	2	3	**4**	**5**	6	7	8	9	10	11	**12**	13	14	15	16	17	18	19	**20**	21	22	23	24	25	**26**	27	28	29	30	31	32
5%	0.5%	20%	5%	8.5	5.9	5.8	**83.4**	**56.4**	4.9	3.9	6.9	5.3	5.8	9.3	**8.6**	6.7	5.0	4.0	6.0	5.7	5.6	5.7	**23.9**	6.6	9.8	5.3	4.7	13.8	**44.4**	13.8	5.4	3.9	6.6	4.0	3.5
10%	1%	20%	5%	5.1	5.2	4.3	**18.0**	**42.7**	4.9	6.3	7.2	4.4	5.1	4.9	**6.8**	6.0	4.8	3.4	5.6	4.9	7.4	5.7	**6.7**	5.5	3.4	4.9	6.1	9.2	**8.4**	6.9	5.5	2.4	5.4	4.8	1.9
15%	2%	20%	5%	6.0	4.4	7.4	**23.4**	**37.0**	8.5	5.0	6.1	2.3	4.4	6.3	**11.4**	7.4	5.7	5.3	2.1	5.5	6.0	7.9	**11.1**	10.5	4.4	6.4	5.9	7.5	**12.1**	11.1	9.8	3.2	5.5	6.0	6.7
20%	5%	20%	5%	5.2	5.7	6.1	**10.3**	**13.7**	5.6	4.9	5.5	5.7	4.5	6.2	**6.4**	5.2	4.7	5.0	5.3	4.0	5.7	5.7	**6.8**	6.9	5.3	4.5	4.8	5.8	**6.2**	7.4	3.3	4.6	5.3	5.1	4.4

It is observed that although the ANN models are capable to detect and quantify the damages, even when testing with noise-free data. It is noticeable that when ANN models are trained with a higher percentage of uncertainty, the PDE values tend to decrease. It is due to reason that with a higher percentage of uncertainty, the range of the smeared Gaussian noise is also wider, resulting in prediction outcomes become less precise. However, the damage existence is only confidently detected using the ANN models trained with 5% uncertainty in mode shapes and 0.5% uncertainty in frequencies. More detailed results are presented in Table 4. Based on the results, when the ANN models are trained with a smaller percentage of uncertainty but tested using a higher percentage of uncertainty, the ANN models are still proficient to detect the damage locations successfully. It is due to the evidence that the PDE values at those damaged segments are higher compared to the other undamaged segments. However, this scenario is not valid for the cases whereby ANN models are trained with a higher percentage of uncertainty and then tested with a lower percentage of uncertainty.

CONCLUSIONS

This paper presented a statistical multi-stage ANN which a limited number of sensors as the input for damage detection. A simulated damage case is tested in the developed ANN model for the feasibility of the proposed method. Different level of uncertainties is introduced in the developed ANN models to examine the influence of the different levels of uncertainty injected in the developed models. With any injection of uncertainty in the model, the proposed method is capable to identify the locations of damage successfully. The proposed multi-stage ANN is also capable to overcome the effect of progressive error in ANN. However, the multi-stage ANN models developed in this study perform better when they are trained with a smaller percentage of uncertainty compared when they are trained with a larger percentage of uncertainty.

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REFERENCES

Bakhary, N., Hao, H., and Deeks, A. J. (2010). Substructuring technique for damage detection using statistical multi-stage Artificial Neural Network. *Advances in Structural Engineering,* **13**(4), 619-639.

Goh, L. D., Bakhary, N., Rahman, A. A., and Ahmad, B. H. (2011). *Application of Neural Network for Prediction of Unmeasured Mode Shape for Damage Detection.* 14th Asia Pacific Vibration Conference, Hong Kong.

Goller, B., and Schueller G.I. (2011). Investigation of model uncertainties in Bayesian structural model updating. *Sound and Vibration,* **330**, 6122-6136.

Lew, J. S. (2008). Reduction of uncertainty effect on damage identification using feedback control. *Sound and Vibration,* **318**, 903-910.

Rosenblueth, E. (1975). Point Estimates for Probability Moments. *Mathematics,* **72**(10), 3812-3814.

Wang, L., Yang, Z., and Waters, T. P. (2010). Structural damage detection using cross correlation functions of vibration response. *Sound and Vibration,* **329**(23), 5070-5086.

Xu, Y. L., Zhang, J., Li, J., and X.M.Wang. (2011). Stochastic damage detection method for building structures with parametric uncertainties. *Sound and Vibration,* **330**, 4725-4737.

Zhang, Q. W. (2007). Statistical damage identification for bridges using ambient vibration data. *Computer & Structures,* **85**, 476-485.