Optimal Sensor Placement for Structural Health Monitoring Using Improved Simulated Annealing

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ABSTRACT

Optimal Sensor placement techniques play a significant role in enhancing the quality of modal data in structural health monitoring particularly for large civil structures, where the large degree-of-freedoms are available for a limited number of sensor. From the literature, fashion of solving this problem has shifted from simple iteration approach to the employment of heuristic algorithms. Although these algorithms are superior in obtaining the global optimal solution, however the major drawback is their high computational cost. Since high efficient heuristic algorithm presents the utmost requirement in such regard, this paper presents an improved encoding method of Simulated Annealing (SA) algorithm for solving the combinatorial optimization problem. Through this algorithm, a coordinate-based solution encoding method is proposed to increase the search performance of SA. The proposed method is tested on a numerical slab model, which consists of two hundred candidates of sensor location. Three types of objective functions are explored in the tests: the determinant of Fisher Information Matrix, Modal Assurance Criterion and Mean Square Error. The results have suggested that the proposed method outperforms the conventional encoding method on the basis of optimizing the objective functions.

KEYWORDS

Optimal Sensor Placement; Simulated Annealing; Coordinate-based Encoding.

INTRODUCTION

Sensor configuration is one of the most important factors to ensure the quality of measured response data in structural health monitoring particularly for vibration-based damage detection. For commonly large civil structures, the measurement at every degree of freedom is not possible due to practical and cost considerations. Therefore, the employment of optimal sensor placement technique in structural health monitoring is essential to obtain reliable data output with a limited number of sensors.

Most of the early sensor placement techniques are through direct ranking method or iterative expansion and elimination approach, which are relatively rapid in determining the optimal solutions compared to the combinatorial optimization technique. However, those solutions are sub-optimal (Yao *et al.*, 1992; Cherng 2003), which have led to incompetence sensor placement solutions. Therefore, many researchers have applied modern heuristic algorithms to optimize the sensor placement problem. For example, Yao et al. (1992) have proposed sensor placement for large space structures using Genetic Algorithm (GA) with forced mutation operator to improve the convergence of fitness function. A number of 187 candidates of sensor placement are considered in the optimization process. The study shows that the GA outperformed the Effective Independence Method (Kammer, 1991) in terms of the fitness function values.

The continuing of this research has extended to civil engineering community. Among the earliest works of optimal sensor placement that is related to fault detection is presented by Xia and Hao (2000). The authors proposed a direct ranking technique for identifying the optimal measurement points, which taking consideration of two factors, namely sensitivity of residual vector to structural damage, and sensitivity damage to measurement noise. Worden and Burrows (2001) investigated different optimization methods for determining the optimal sensor placement for a cantilever plate. Twenty candidates of sensor location were considered in the optimization process. They concluded that optimization using SA is slightly better than GA in terms of the quality of sensor distribution.

Since then, many improvements have been made in terms of algorithm's performance. Guo *et al.* (2004) developed a new constraint to improve the convergence trends of fitness function. Rao and Anandakumar (2007) introduced hybrid Particle Swarm Optimization to solve the sensor placement problem. Liu *et al.* (2008) designed forced mutation GA with two-dimension array solution coding method on a spatial lattice structure. Yi *et al.* (2011) chose high-rise structure for sensors placement optimization via generalized GA with dual-structure solution coding method. However, in most early studies, the number of sensor locations is relatively small, thus the large scale of this problem is not well assured. Furthermore, most of the works developed are GA-based technique while efforts of fine-tuning SA or other types of optimization method are quite limited.

Since an efficient optimization method is always demanded due to the trade between qualities and computational efforts, in this paper, the sensor placement problem is formulated as a combinatorial optimization problem and solved via SA algorithm to remedy the sub-optimal issue as mentioned earlier. In order to test the algorithm's performance in larger scale, two hundred of DOFs are selected from a slab model as the candidates of sensor locations. Determinant of Fisher information matrix (FIM), modal assurance criterion matrix (MAC) and sum of mean square error (MSE) are applied in the optimization as the objective functions. To improve the search performance of SA for sensor placement, a new encoding method is proposed to allow the search process in higher dimension instead of one dimension in conventional encoding methods. The performance of the proposed method is compared with the conventional encoding method through the three objective functions. It is demonstrated that the proposed method is able to enhance the search performance of SA in both quality of sensor configuration and convergence speed of objective function value. A comparison is also conducted for the objective functions in order to evaluate their reliability in sensor placement optimization.

BACKGROUND

The concept of SA in combinatorial optimization can be found in Metropolis *et al.* (1953), Kirkpatrick *et al.* (1983) and Ingber (1989). Conceptually, the idea of SA is analogous to the physical process of annealing, where the material is heated to its melting point and followed by slow cooling process to recrystallize its microstructure formation and thus eventually improve the material properties. Therefore, on a similar account, the final goal of SA algorithm is to find a global optimal solution corresponded to a problem which governed by an objective function.

The algorithm begins with a predefined initial sensor configuration. Based on this input, the algorithm then randomly finds a new configuration around its neighbours, where the distance of random search is proportional to the annealing temperature. In this study, the new generated solution is constrained by a set of conditions such as the upper and lower bounds of sensor locations. Next, this new solution is compared to the previous configuration and the good solution is always accepted, while the weaker one is accepted if only a random generated number [0, 1] is greater than a probability value which is given by the Boltzmann distribution;

$$p = exp\ (-\Delta E/\ T) \tag{1}$$

where ΔE is the change in objective function value and T is the temperature control parameter which has the same unit as the objective function. The temperature parameter will be gradually reduced and these steps are repeated until the function value has converged or reach maximum number of iteration that is defined by the user.

IMPROVED ENCODING METHOD

The common type of coding for sensor placement problem is in binary format as shown in Table 1 where n_l denotes the total number of potential location. For instances, it is applied in Worden and Burrows (2001), Guo *et al.* (2004) and Liu *et al.* (2006). Another alternative is by using sensors decimal coding method. Table 2 shows the configuration of the method with n_s denotes the total number of sensor. The application of decimal coding method can be found in Rao and Anandakumar (2007) and Liu, *et al.* (2008). However, since these encoding methods are in one dimensional format, the search space of SA is only limited to either in longitudinal directions (positive or negative) or transverse directions (positive or negative), which may lead to a longer time of function convergence and a local optimal solution.

Table 1. Binary sensor placement codes.

Location	1	2	3	4	5	6	7	8	…	n_l
Availability	0	1	0	1	0	0	0	0	…	1

Table 2. Decimal sensor placement codes.

Sensor	1	2	3	4	5	6	7	8	…	n_s
DOF	7	8	11	12	42	44	46	75	…	79

When applying SA in an optimization problem, the design of the neighborhood structure is of high importance because the neighborhood ranges can significantly affect the accuracy of the solutions (Miki et al., 2006). In optimal sensor placement problem, a coordinate-based solution encoding method is therefore proposed to overcome a lesser spatial information issue as encountered in conventional one dimensional encoding method. The basic idea of the proposed method is to allow the random search of sensor points to be based on the real geometry of the given structure. This is done by acquiring extra coordination parameters (x_i, y_i, and z_i) to each individual solution as illustrated in figure below.

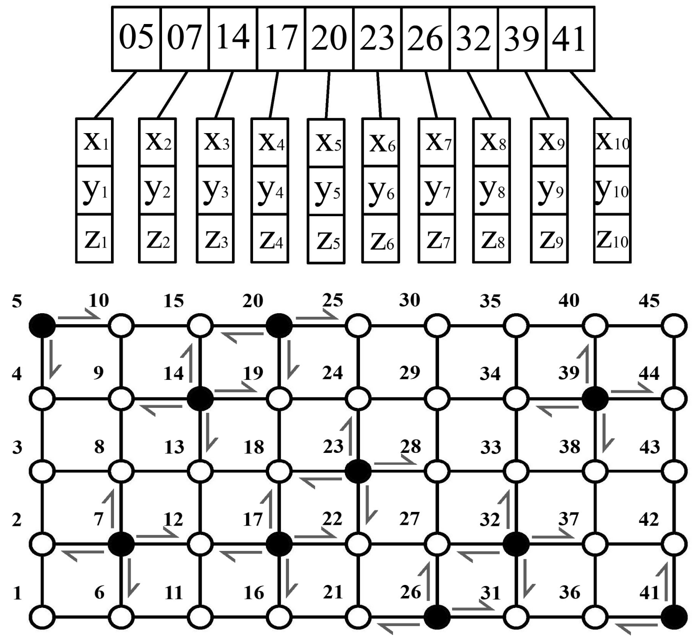

Figure 1. The proposed coordinate-based encoding method.

Figure 1 illustrates an initial sensor configuration which placed at nodal points 5, 7, 14, 17, 20, 23, 26, 32, 39 and 41, respectively. Since the performance of SA depends on the search directions, this method imposes the random search based on a global coordinate system at the three major axes. Nevertheless, in this paper, only two axes (x_i, y_i) of search directions are demonstrated for the analytical model, while z_i is kept constant. The random numbers are assigned in every axis' parameters independently to provide flexible movement of sensor points during the optimization process. This allows the sensor points to move in all directions simultaneously. Dissimilar to the conventional way, in which the random number is assigned in single direction, either in longitudinal direction or transverse direction depending on the sequence of nodal numbering.

OBJECTIVE FUNCTIONS

In this paper, three objective functions are used as the sensor placement strategy. The first criterion is based on the Fisher Information Matrix (FIM) of mode shape. This information matrix is derived from the covariance matrix of estimated error in which the parameter estimates is inversely proportional to the information matrix (Kammer and Yao, 1994). Briefly, the information matrix indicates a summation of the contribution of each degree of freedom of the structure. Therefore, the best estimate of damage coefficients of sensor distribution can be obtained by maximizing the determinant of FIM (Guo et al., 2004). This measurement is commonly simplified by eliminating the noise parameters, assuming that every sensor possesses the same noise property. Thus, the Fisher information matrix used is written as

$$Q = \varphi^T \varphi \quad (2)$$

where φ represents the modal vectors partitioned to the sensor locations and superscript T represents the transpose operation of matrix. Hence, the first objective function used in this paper is to maximize the determinant of Q, which is given as equation (3) below.

$$FIM = Det.|\varphi^T \varphi| \tag{3}$$

The second objective function used is for the maximization of the sum of modal assurance criterion between full-resolution finite element generated mode shapes, φ_A and sensor placement estimated mode shape, φ_B using bicubic interpolation which given in equation (4), where *m* denotes the number of target modes.

$$MAC = \sum_1^m \frac{|\varphi_A^T \cdot \varphi_B^T|^2}{\varphi_A^T \varphi_A \cdot \varphi_B^T \varphi_B} \tag{4}$$

The third objective function used in this paper is the sum of mean square error (RMS) of modes shapes which written in equation (5), where *n* represents the total number of sensor location.

$$MSE = \sum_1^m \frac{\sum_1^n |\varphi_A - \varphi_B|^2}{n} \tag{5}$$

NUMERICAL MODEL

A numerical model of a rectangular concrete slab is used to demonstrate the efficiency of the proposed method. The dimensions of the slab are 7.0m (length), 2.7m (width) and 0.2m (thickness). The assigned material properties are, elastic modulus, E = 22.5 GPa; mass density, ρ = 2450 kgm^{-3}; poison's ratio, r = 0.15. The model consists of 196 shell elements and 232 nodes. By excluding 16 DOFs from the simply supports and 16 DOFs from the shorter ends, thus a number of 200 nodes are selected as the candidates of sensor locations. For this purpose, only the first three vertical bending modes are considered in the analysis as illustrated in Figure 2. The natural frequencies are 41.95Hz, 164.24Hz and 357.44Hz respectively.

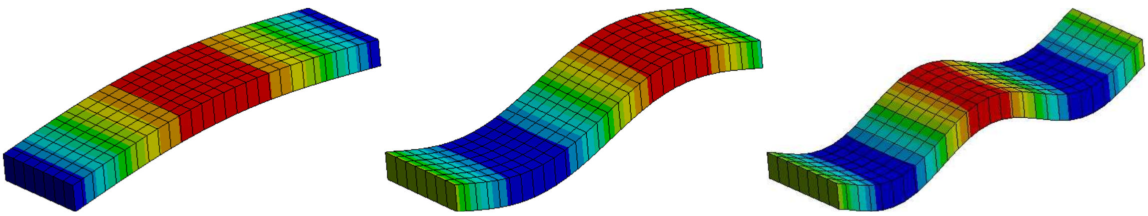

Figure 2. The first three vertical bending modes.

RESULTS AND DISCUSSION

The performance of SA using the proposed encoding method and a conventional one dimensional method are compared. Figure 3 shows the comparisons between decimal encoding method (SA1) and the proposed method (SA2) using three different objective functions respectively. In these tests, ten sensor locations are randomly selected as the initial input and a 0.99 of cooling factor is applied. Based on the results, it is observed that the proposed method provides better convergence value as compared to the decimal encoding method. For example, in Figure 3(a), the FIM values are higher in most of the iterations especially after 250 iterations. As expected, the position-based random search enables the algorithm to locate the optimal sensor points more efficiently. Similar result is shown in Figure 3(b) where comparison for both encoding methods using the sum of MAC values over the target modes is presented. As the iteration proceeds, it is seen that the proposed method offers a better objective function value in most of the iterations. Here, a higher in MAC value indicates that the corresponding solution is more accurate in identifying the modal vectors. On a different measure, the same performance is exhibited in Figure 3(c) where the sum of mean square error between two mode shape plots is minimized. It is clear that the MSE function values of the proposed encoding method are significantly lower compared to the decimal encoding method.

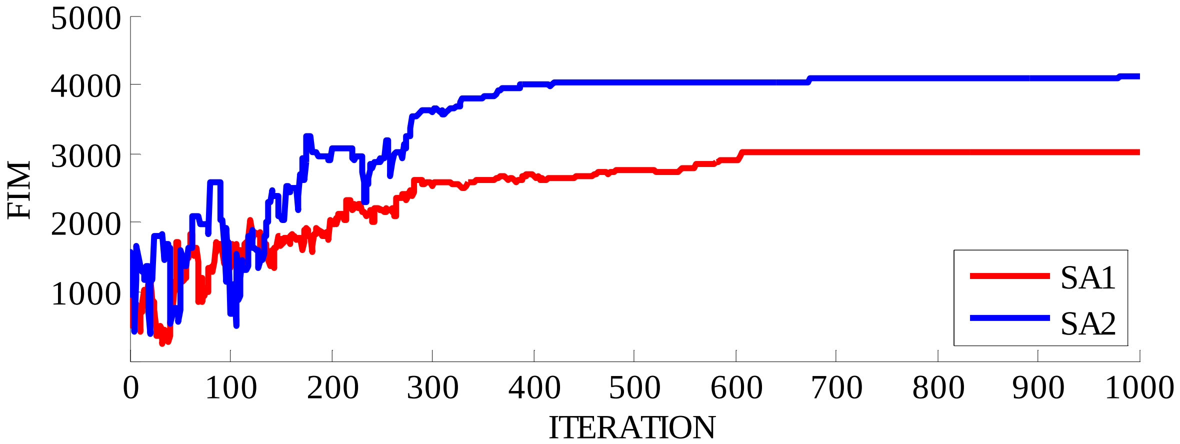

Figure 3(a): Maximization the of determinant of Fisher information matrix (FIM)

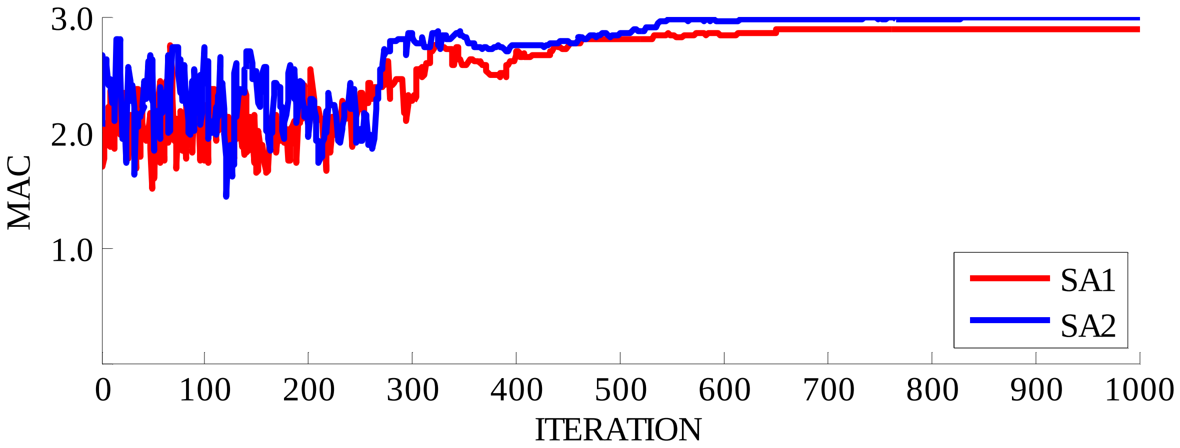

Figure 3(b): Maximization of the total modal assurance criterion (MAC)

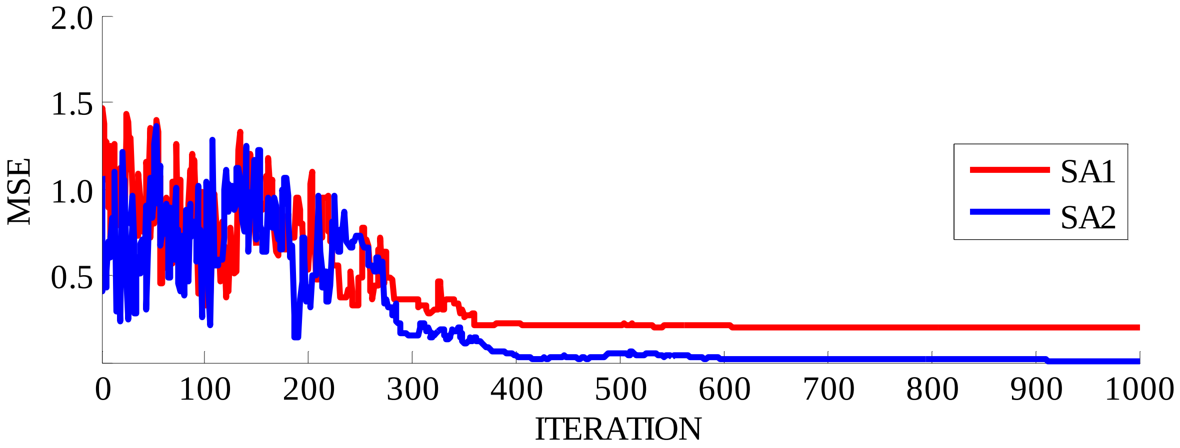

Figure 3(c): Minimization of the total mean square error (MSE)

In the optimization process, greater number of sensors mostly offers better objective function value and requires longer time of convergence. Thus, the optimal sensor locations for 10, 15 and 20 measured points placed on the model to identify the modal parameters are shown in Tables 3-5. By comparing the total squared errors (TSE) values, it is seen that the sensor placements produced by the FIM function contains larger errors compared to those produced by MAC and MSE functions. This observation may due to the nature of FIM function itself, which places the sensors in within a short distance or interval on the structure.

Table 3. Comparison of optimal sensor locations for 20 measured points.

Function (Value)	Sensor Locations	TSE
FIM (2.9351e+04)	33 39 40 41 42 43 47 48 49 96 97 98 103 104 105 153 154 159 160 161	15.99
MAC (2.9972)	4 10 17 21 33 40 44 51 65 80 97 112 131 152 153 157 176 177 179 192	0.66
MSE (0.0045)	9 23 33 40 52 57 64 70 71 74 76 81 91 97 104 113 152 161 184 186	0.96

Table 4. Comparison of optimal sensor locations for 15 measured points.

Function (value)	Sensor Locations	TSE
FIM (1.3868e+04)	40 41 42 47 48 96 97 103 104 105 153 154 160 161 168	16.88
MAC (2.9971)	15 17 40 41 72 83 89 104 118 121 144 148 161 176 179	0.77
MSE (0.0048)	22 25 40 53 57 80 81 85 105 112 140 161 165 168 188	1.03

Table 5. Comparison of optimal sensor locations for 10 measured points.

Function (value)	Sensor Locations	TSE
FIM (4.1198e+03)	40 41 48 96 97 104 105 153 160 161	15.52
MAC (2.9921)	11 33 40 73 96 105 128 160 161 192	1.94
MSE (0.0071)	17 40 49 81 88 105 120 145 168 177	1.54

CONCLUSIONS

This study presents an improved encoding method in sensor placement technique using SA algorithm. Moreover, the results of sensor placements provided by three different objective functions namely, determinant of Fisher information matrix (FIM), modal assurance criterion, and mean square error (MSE) are studied. The developed position-based encoding method has proven to be superior in terms of optimizing aforementioned objective functions comparatively to the conventional one dimensional method. The study also concluded that MAC and MSE functions have significantly less estimation errors in modal identification relative to FIM function.

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REFERENCES

Cherng, A. P. (2003) Optimal Sensor Placement for Modal Parameter Identification Using Signal Subspace Correlation Techniques, *Mechanical Systems and Signal Processing* **17**(2) 361-378.

Guo, H. Y., Hang, L. Z., Zhang, L. L., and Zhou, J. X. (2004) Optimal Placement of Sensors for Structural Health Monitoring Using Improved Genetic Algorithms, *Smart Materials and Structures* **13**(3) 528-534.

Ingber, L. (1989) Very Fast Simulated Re-annealing, *Mathematical and Computer Modelling* **12**(8) 967-973.

Kammer, D. C. (1991) Sensor Placement for On-orbit Modal Identification and Correlation of Large Space Structures, *Journal of Guidance, Control and Dynamics* **14** 251-259.

Kammer, D. C. and Yao, L. (1994) Enhancement of On-orbit Modal Identification of Large Space Structures Through Sensor Placement, *Journal of Sound and Vibration* **171**(1) 119-139.

Kirkpatrick, S., Gelatt, C. D., Jr. and Vecchi, M. P. (1983) Optimization by Simulated Annealing, *Science* **220**(4598) 671-680.

Liu, W., Gao, W. C., Sun, Y. and Xu, M. J. (2008) Optimal Sensor Placement for Spatial Lattice Structure Based on Genetic Algorithms, *Journal of Sound and Vibration* **317**(1-2) 175-189.

Liu, W., Hou, Z. K. and Demetriou, M. A. (2006) A Computational Scheme for The Optimal Sensor and Actuator Placement of Flexible Structures Using Spatial Measures, *Mechanical Systems and Signal Processing* **20**(4) 881-895.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N. and Teller, A. H. (1953) Equation of State Calculations by Fast Computing Machines, *Journal of Chemical Physics* **26**(6) 1087–1092.

Miki, M., Hiwa, S. and Hiroyasu, T. (2006) Simulated Annealing using an Adaptive Search Vector, *Proceedings IEEE Conference on Cybernetics and Intelligent Systems* pp. 1-6.

Rao, A. R. M. and Anandakumar, G. (2007) Optimal Placement of Sensors for Structural System Identification and Health Monitoring Using a Hybrid Swarm Intelligence Technique, *Smart Materials and Structures* **16** 2658–2672.

Worden, K. and Burrows, A. P. (2001) Optimal Sensor Placement for Fault Detection, *Engineering Structures* **23**(8) 885-901.

Xia, Y. and Hao, H., (2000). Measurement Selection for Vibration-based Structural Damage Identification. *Journal of Sound and Vibration* **236** (1) 89-104.

Yao, L. T., Sethares, W. A. and Kammer, D. C. (1992) Sensor Placement for On-orbit Modal Identification of Large Space Structure via a Genetic Algorithm, in: *Proceedings IEEE International Conference on Systems Engineering* pp. 332 - 335.

Yi, T. H., Li, H. N. and Gu, M. (2011) Optimal Sensor Placement for Health Monitoring of High-Rise Structure Based on Genetic Algorithm, *Mathematical Problems in Engineering* **2011**(2011).