# An experimental study on the elasto-plastic dynamic increase factor of sudden support-loss response

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## ABSTRACT

Dynamic increase factors (DIFs) have been used for considering the dynamic effect in nonlinear static progressive analyses. In this study, a small-scale test setup is devised to investigate the effect of material nonlinearity on the DIF under sudden support loss. Displacement-based, force-based, and neutral-based DIFs are defined. Displacement DIFs are obtained from the measured displacement response. Force DIFs are calculated from the nonlinear static and dynamic load-displacement response curves. Neutral DIFs are obtained from dividing the maximum dynamic displacement by the neutral displacement. Test results indicate that the displacement DIF may increase with ductility at the onset of inelastic range and then start to decrease after certain greater ductility. A different variation is observed for the force DIF. It may decrease with ductility at the onset of inelastic range. However, the decreasing trend is reversed as the ductility increases further. As for the neutral DIF, it may asymptotically decrease to 1.0 with increasing ductility. This study reveals that the neutral DIF cannot be used to account for the dynamic effect on either the displacement or force response in the plastic range. Also, both the displacement and force DIFs may be predicted by using analytical pseudo-static formulations.

## KEYWORDS

Dynamic increase factor; small-scale test; support loss.

## INTRODUCTION

Detailed step-by-step procedures for static progressive collapse analysis have been issued by US General Service Administration (GSA) (2003) and recommended in the Unified Facilities Criteria UFC 4-023-03 by the Department of Defense (DoD) (2005, 2009). The recommended approaches are based on immaculate member removal and have been widely accepted in practical engineering. Dynamic effect was simulated with a constant dynamic amplification factor equal to 2.0 in the static analysis procedures. Some studies revealed that the constant amplification factor of 2.0 may lead to inconsistent results with those obtained using nonlinear dynamic analysis (Ruth *et al*., 2006; Tsai and Lin, 2008; Kim and Kim, 2009). In the latest issued UFC 4-023-03 guidelines (DoD, 2009), two different magnification factors, namely the load increase factor (LIF) and dynamic increase factor (DIF), are suggested. The LIFs are used to account for both the material nonlinearity and dynamic effect in linear static analysis, while the DIFs are for the dynamic effect in nonlinear static analysis. Empirical formulae established on nonlinear dynamic analysis results of various frame models were provided for estimation of the LIF and DIF in the guidelines (Marchand *et al*., 2009). Also, analytical expressions of DIFs based on elasto-plastic, single degree-of-freedom (SDOF) models were proposed around the same period (Tsai, 2010). Some dynamic tests have been performed to investigate the structural response under sudden support loss. An experimental study on the dynamic effects in progressive failure of structures was carried out with a spoked-wheel structure (Pretlove *et al*., 1991). It indicated that plasticity could blunt the sensitivity of a structure to dynamic effect in progressive failure. Recently, prototype column-loss experiments were conducted to investigate the dynamic behavior of gravity load redistribution (Sasani *et al*., 2007; Sasani and Sagiroglu, 2010). Due to the nature of failure tests, usually only dynamic response under a given loading magnitude could be obtained. The DIF was then estimated from the measured time-history response in the dynamic failure test (Matthews *et al*., 2007; Tian and Su, 2011).

In this study, a small-scale test setup devised to investigate the effect of plasticity on the DIF under sudden support loss is presented. Static and dynamic support-release tests are carried out manually. On the basis of three different definitions, experimental DIFs under various ductility demands are obtained from the measured static and dynamic responses. Differences of the three definitions and their practical implications are discussed. Pseudo-static response of the test specimen, which is a load-displacement curve constructed from the nonlinear static response, is used to capture the variation of DIFs with ductility demand. With consideration of both post-yield stiffness ratios and ductility, analytical formulations are also applied to estimation of the DIFs.

## TEST SETUP AND RESULTS

### Test setup

Due to limited resources, a small-scale test setup with manual support-release mechanism has been devised for the DAF experiment. Fig. 1 shows a schematic drawing for the elevation view of the test setup. The test specimen was made of structural steel with design yield stress of 400 MPa and had a cross section of 30 mm wide and 3 mm depth. From material tests, the measured yield stress and strain were equal to 422 MPa and 0.002, respectively. For installation of the test specimens, L40×40×3 steel angles were welded to both ends. Clear length of the specimen was equal to 270 mm. Then, one end was bolted to an I100×75×5×8 steel column and the other to a transfer connection. The transfer connection served as the loading point and was attached with an accelerometer. For each specimen, three strain gauges were disposed 20 mm away from the weld connection, as shown in the figure. A laser displacement meter was installed to a reference frame to measure the displacement at a distance of 225 mm from the presumed fixed end. A hanger, used to support the imposed loading before sudden release, was fixed to the reference frame with a pinned connection. The hanger could be manually knocked off by a hammer to simulate the sudden loss scenario. A rectangular steel basket was connected to the transfer connection and used to accommodate the imposed loadings. Iron blocks with average weight of 6.30 N and lead plates with average weight of 12.37 N each were used as the imposed loadings. The picture of a deformed specimen under static loading is shown in Fig. 2.

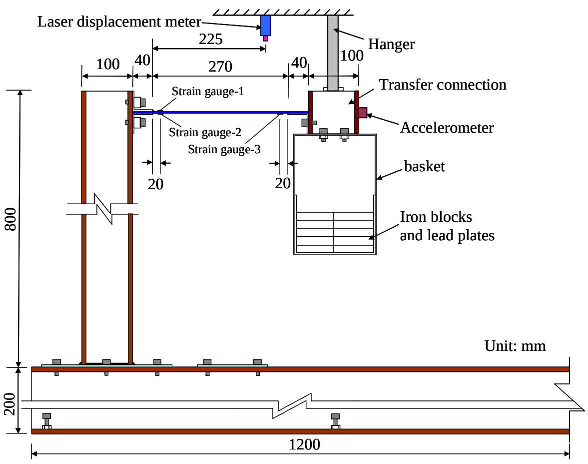


**Figure 1.** A schematic drawing of the test setup

**Figure 2**. A photo of a loaded specimen

### Test results

Measured nonlinear static, maximum dynamic, and neutral responses are compared in Fig. 3. In the figure, the ordinate is the applied loading and the abscissa is the displacement measured with the laser sensor. The nonlinear static response was obtained from the statically loading and unloading test of one specimen. Its estimated yield displacement is 38.1 mm, of which the associated yield load is 78.69 N. The maximum dynamic and neutral responses were obtained from the recorded displacement time histories of sudden support-loss tests, as indicated in Fig. 4, where three selected time histories are displayed. The displacement time histories reveal that the ratio of maximum to neutral response decreases with increasing loading. Sudden support-loss tests were carried out for eleven specimens in total for the dynamic and neutral response curves. The neutral response is consistent with the static response in the elastic range. Both of the neutral and static displacements are in fact equal to half of the maximum dynamic displacement under small loadings. However, as the specimen is forced into the plastic range, the neutral response curve starts to divert from the static curve, as observed from Fig. 3. An approximately parallel offset from the dynamic envelope is observed for the inelastic neutral response. In addition, a pseudo-static response curve, which was calculated from the accumulated strain energy below the nonlinear static response curve divided by the corresponding displacement, is included in the figure. It is seen that the pseudo-static response is approximated to the maximum dynamic response under support loss. This reveals that the pseudo-static analysis technique may be used in estimation of the DIFs, which will be explained later.

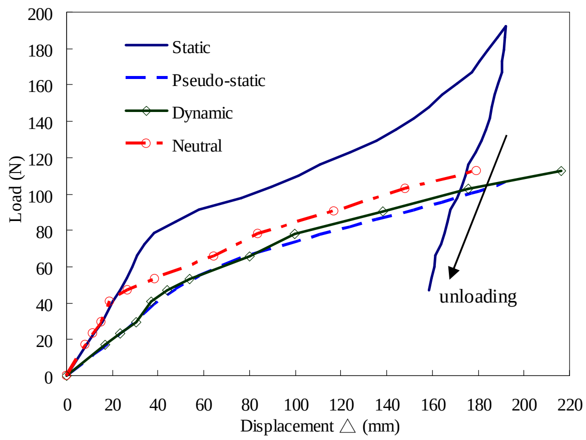


**Figure 3.** Load-displacement response curves

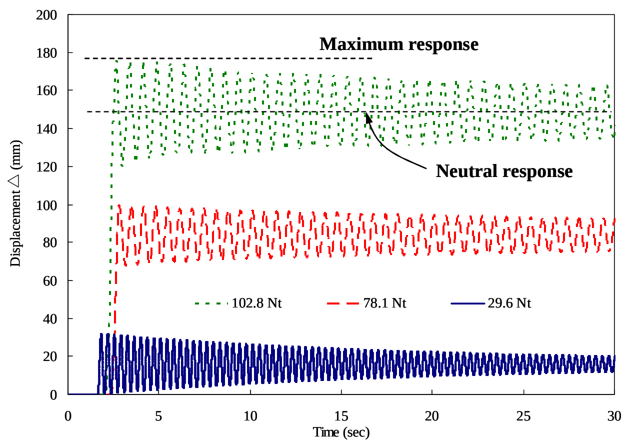


**Figure 4**. Typical displacement time histories

## DYNAMIC INCREASE FACTOR (DIF)

### Definitions

The dynamic amplification factor is conventionally calculated as the ratio of maximum dynamic displacement $\Delta_d$ to static displacement $\Delta_{st}$ for an elastic SDOF model under an applied loading $P = P_d$, as shown in Fig. 5. Therefore, a displacement-based DIF may be defined as

$$DIF_\Delta = \Delta_d / \Delta_{st} \qquad (1)$$

According to this definition, the experimental $DIF_\Delta$ may be calculated directly by dividing the dynamic envelope by the corresponding static response because the static and dynamic tests are conducted with load control technique.

Conversely, as observed from Fig. 5, the elastic DIF is also equal to the ratio of static force to dynamic force under an equal displacement demand. Thus, a force-based DIF may be defined as

$$DIF_P = P_{st} / P_d \qquad (2)$$

where $P_{st}$ and $P_d$ are respectively the required static and dynamic force under the same deflection, $\Delta_{st} = \Delta_d$. Apparently, the displacement- and force-based DIFs are equal in the elastic range. For experimental $DIF_P$, the dynamic force and displacement are provided from the support-release tests. However, the required static force under the same displacement demand has to be determined from linear interpolation of the static load-displacement response curve.

In addition, a common approach for calculating the DAF from measured dynamic time histories is to divide the maximum response by the neutral response (Matthews *et al*., 2007; Tian and Su, 2011). For comparison, it is defined as neutral DIF. The neutral DIF is usually obtained from the displacement time history as

$$DIF_N = \Delta_{\max} / \Delta_{nul} \quad (3)$$

where $\Delta_{\max}$ and $\Delta_{nul}$ are the maximum and neutral displacement, respectively. In this way, the DIF may be determined from a single dynamic test without the need of static test. It is noteworthy that for an elastic SDOF model subjected to sudden support loss, the displacement-based, force-based, and neutral DIFs are all equal to 2.0. However, certain differences among the three definitions may arise if inelastic response is induced under support loss.

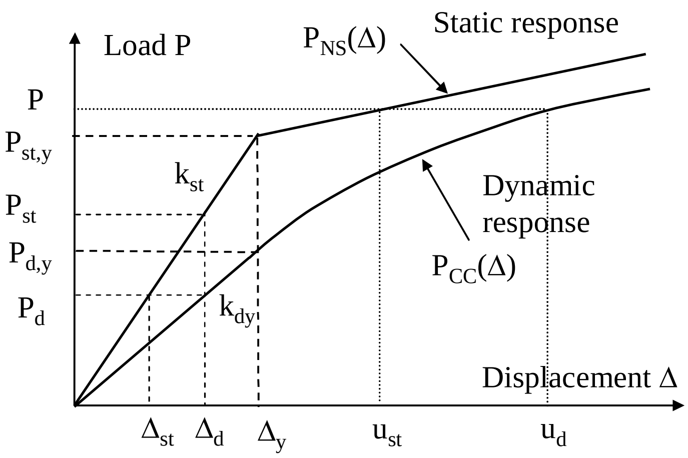


**Figure 5** Explanation of the definition of displacement- and force-based DIFs

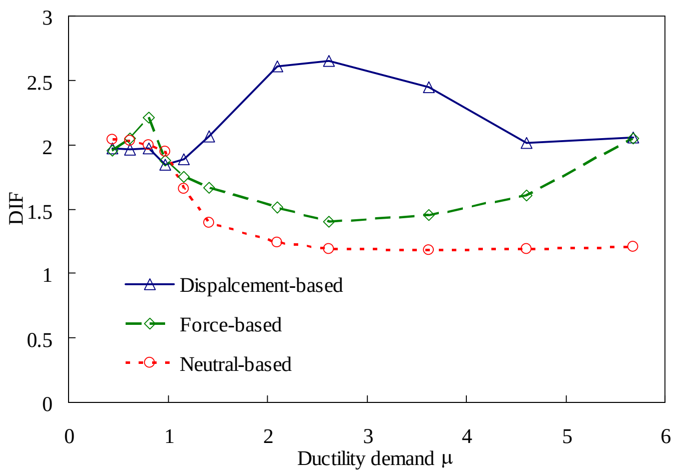


**Figure 6** Comparison of different DIFs

## Comparison

Fig. 6 presents the DIFs calculated based on the aforementioned three definitions using the measured displacement response. The abscissa is expressed as ductility demand, which is obtained by dividing the maximum displacement by the yield displacement 38.1 mm. It is seen that the three definitions have consistent predictions only in the elastic range. The displacement-based DIF may increase with ductility demand at the onset of inelastic phase and then start to decrease after certain larger ductility. Eventually, it decreases to around 2.0 as the ductility demand is greater than 4.5. This reveals that the support-loss behavior of the specimen appears to be solely governed by its post-yield stiffness under large deformation such that the inelastic DIF is approximated to the elastic DIF. On the contrary, a different variation for the force-based DIF is observed from the figure. It is seen that it decreases with ductility demand at the onset of inelastic phase. However, due to the strain hardening effect, the decreasing trend is reversed as the ductility increases further. Similar to the displacement-based DIF, it may return to 2.0 under larger ductility demand. As for the neutral DIF, it may asymptotically decrease toward 1.0 with increasing ductility. Since the neutral response cannot represent the nonlinear static behavior, it is thus not suitable for quantifying the dynamic effect on either the displacement or force response in the plastic range. The neutral DIF can only make sense for elastic vibration. In practice, the force- and displacement-based DIFs may be applied to different situations. If the collapse resistance of building structures subjected to column loss is of concern, the force-based DIF may be used in the nonlinear static pushdown analysis. Alternatively, the displacement-based DIF may be used in prediction of the maximum displacement response if the structural performance under column loss is desired.

## Prediction

Both the displacement- and force-based DIFs may be calculated from the numerical pseudo-static response as long as the nonlinear static load-displacement curve is given. Normalized pseudo-static response is used to calculate the numerical displacement- and force-based DIFs, as shown in Figs. 7 and 8, respectively. It is seen that the numerical DIFs are consistent with the experimental DIFs. The DIFs calculated from the empirical formula recommended by UFC 4-023-03 (DoD, 2009) for steel frames are also shown in Fig. 7. Due to the contribution of post-yield stiffness, the empirical formula appears to significantly underestimate the force-based DIFs as the ductility is larger than 3.

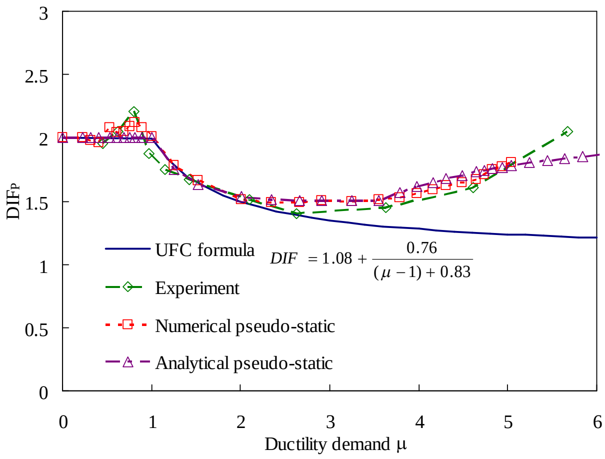


**Figure 7** Prediction of force-based DIFs

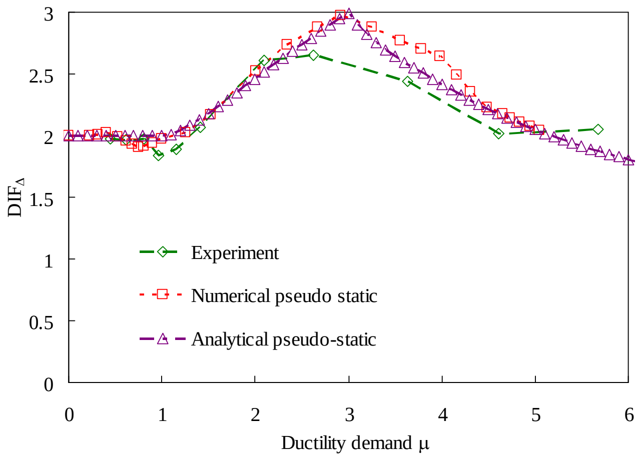


**Figure 8** Prediction of displacement-based DIFs

In practical engineering, the nonlinear structural behavior is usually modeled by piecewise linear approximation. If the static load-displacement curve of the test specimen is idealized as tri-linear behavior, the elastic, post-yield, and hardening stiffness are estimated as 20.65 N/cm, 5.19 N/cm, and 11.07 N/cm, respectively. The post-yield and hardening stiffness ratios of load and displacement response for the tri-linear approximation are respectively calculated as

$$\alpha_1 = (P_s - P_{st,y})\Delta_y / (\Delta_s - \Delta_y) / P_{st,y} \quad (4a)$$
$$\alpha_2 = (P_u - P_s)\Delta_y / (\Delta_u - \Delta_s) / P_{st,y} \quad (4b)$$

In the above equations, $P_{st,y}$, $P_s$ and $P_u$ are respectively the yield loading, the loading at the onset of strain-hardening, and the maximum applied loading in the static test. Their corresponding displacements are denoted as $\Delta_y$, $\Delta_s$, and $\Delta_u$. Analytical formulae of displacement- and force-based DIFs have been derived from pseudo-static analysis for idealized elasto-plastic models (Tsai, 2010). Similar efforts are made to derive analytical expressions for the idealized tri-linear response. The analytical displacement- and force-based DIFs are shown in Figs. 7 and 8, respectively. With proper consideration of the post-yield and hardening stiffness ratios, it is seen that the analytical pseudo-static methodology may approximately capture the variation of DIFs with ductility demands. The analytical approach appears to be an effective option for predicting the DIFs in progressive collapse analysis. Because the maximum ductility obtained from the static loading test is 5.04, plastic deformation larger than that is not considered in the estimation of the hardening stiffness ratio $\alpha_2$. Hence, more significant discrepancy between the analytical and experimental DIFs is observed as the ductility demand is larger than 5.0.

## CONCLUSIONS

A small-scale test setup was devised to investigate the inelastic dynamic increase factors (DIFs) for structures subjected to sudden column loss. Strip-type steel specimens were designed to be fixed at one end and supported by a hanger at the other. The hanger could be manually knocked off by a hammer to simulate the sudden loss scenario. Based on three different definitions, experimental DIFs were determined from static and dynamic support-release tests. The following findings are obtained from the test results.

(1) The DIFs estimated from the neutral response decreases asymptotically toward 1.0 with increasing ductility. They cannot properly account for the inelastic dynamic effect on either the displacement or force response. The displacement-based DIFs are generally larger than two and exhibit a concave downward variation with ductility demand. On the contrary, the force-based DIFs are generally less than two and exhibit a concave upward variation with ductility demand. Both approach the elastic DIF as the test specimen is loaded into the strain-hardening range.

(2) Without appropriately considering the influence of post-yield stiffness ratios, the empirical formula recommended by UFC 4-023-03 guidelines fails to capture the variation of the DIFs as the ductility demand is larger than 3. In such a case, the dynamic effect may be underestimated with the empirical formula.

(3) The pseudo-static analysis technique may be applied to prediction of the inelastic displacement- and force-based DIFs. With proper consideration of the post-yield and hardening stiffness ratios, the analytical inelastic DIFs may be approximated to the experimental results.

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