Responses to comments from reviewers (Please see black texts):

Comments from Reviewer 1:
Accepted for publication in its current form

Comments from Reviewer 2:
Accepted for publication with minor revision. The details of the comments are as follows:

This paper discusses a simple method to calculate natural frequencies of a bilinear system, in particular, a fatigue crack. The proposed method divides a measured free vibration signal into two components: above the zero-axis and below the zero-axis. The natural frequencies are calculated independently for each component. This method simplifies a non-linear problem into a combination of linear problems. A single degree-of-freedom system was considered as a verification model.
Comment:

(1) In the last paragraph of page 3, Eq. (1a), (1b) should be Eq. (2a), (2b), respectively.

Responses from authors: Thanks for pointing this out. This has been fixed.

(2) In Table 1, an explanation of how the frequencies of x<0 and x>0 was calculated for "Analytical" case will be required. As the earlier paragraph stated that it was obtained by eigensolution of the mass and stiffness matrix, how come a determined mass and stiffness matrix can yield different results for x>0 and x<0 case?

Responses from authors: Table 1 is as below:

Table 1. Identified natural frequency of the bilinear oscillator (Hz)

	Analytical	Identified results		
		without noise	5% noise	10% noise
From global responses	22.84	22.84	22.84	22.84
From the responses $x>0$	19.49	19.49	19.45	19.45
From the responses $x<0$	27.57	27.47	27.65	27.78

The equation of motion of the system under free vibration can be expressed as

$$\begin{cases} m\ddot{x} + c_1\dot{x} + k_1 x = 0 & x \geq 0 \\ m\ddot{x} + c_2\dot{x} + k_2 x = 0 & x < 0 \end{cases} \quad (1)$$

As discussed in the paper, all positive responses (the responses $x>0$) are calculated from the first equation of Eq. (1) (or Eq. (2a)), which is associated with the stiffness of k_1. And all negative ones (the responses $x<0$) are calculated from the second equation of Eq. (1) (or Eq. (2b)), which is associated with the stiffness of k_2. k_1 is not equal to k_2 in a bilinear system. Accordingly, the frequency extracted from the responses $x>0$ and the one extracted from the responses $x<0$ are not equal.

A novel system identification approach for bilinear systems

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ABSTRACT
Nonlinear phenomena are widely encountered in practical engineering applications. A typical example is fatigue cracks which open and close under dynamic loadings, exhibiting a breathing characteristic and introducing bi-linearity into structures. To detect fatigue cracks, a simple and efficient system identification method is proposed for bilinear systems in this paper. This method takes best advantage of dynamic characteristics of bilinear systems. The basic idea of this method is to identify the parameters in each stiffness region separately by dividing the global responses into different parts representing the corresponding stiffness regions according to the stiffness interface, which transfers nonlinear system identification into linear system identification. The proposed system identification approach has been successfully validated by numerical simulations.

KEYWORDS
System Identification; Bilinear Systems; Fatigue Cracks.

INTRODUCTION

Nonlinear phenomena are widely encountered in practical engineering applications. A typical example is fatigue cracks which open and close under dynamic loadings, exhibiting a breathing characteristic and introducing bi-linearity into structures. The fatigue cracks, if undetected, could lead to a catastrophic failure of the whole structure (Qian *et al.*, 1990; Chu and Shen, 1992; Worden and Tomlinson, 2001).

A number of approaches have been proposed to detect fatigue cracks. They can be classified into local detection approaches and global detection approaches. Among local detection approaches, lamb waves (Leong *et al.*, 2005; Ihn and Chang, 2004) and laser ultrasound (Shan and Dewhurst, 2003) as well as acoustic emission techniques (Roberts and Talebzadeh, 2003) have been explored to detect fatigue cracks. However, expensive equipment may be involved when using these methods. Therefore, global approaches based on nonlinear dynamic behaviors of fatigue cracks have attracted considerable attention. For example, Rivola and White (1998) detected the existence of a fatigue crack by means of bispectral analysis, which was a subset of higher-order statistical analysis; Surace *et al.* (2001) applied higher order Frequency Response Functions (FRFs) which are based on the Volterra series to detect cracks in beam-like structures; Loutridis *et al.* (2005) proposed a crack identification method based on the instantaneous frequency which was obtained by performing Hilbert transform on the obtained modes using the empirical mode decomposition of responses. However, these approaches are either insensitive or sophisticated.

In this study, to detect fatigue cracks, a simple and efficient system identification method is developed based on dynamic characteristics of systems with fatigue cracks. The central idea of the proposed method is as follows: the responses associated with different stiffness regions are first separated from one another, and then the natural frequencies in each region can be identified from the respective separated responses using any approach proposed for linear systems. In this way, the problem of nonlinear system identification is transferred into that of linear system identification. By comparing the natural frequency in each region, fatigue cracks can be easily identified and quantified. This paper is focused on the system identification approach due to the limit of length. This remainder of the paper is organized as follows. First, a system identification method for bilinear systems is proposed. Then, the performance of the proposed method is demonstrated by numerical simulations.

SYSTEM IDENTIFICATION THROUGH SEPARATING GLOBAL RESPONSES

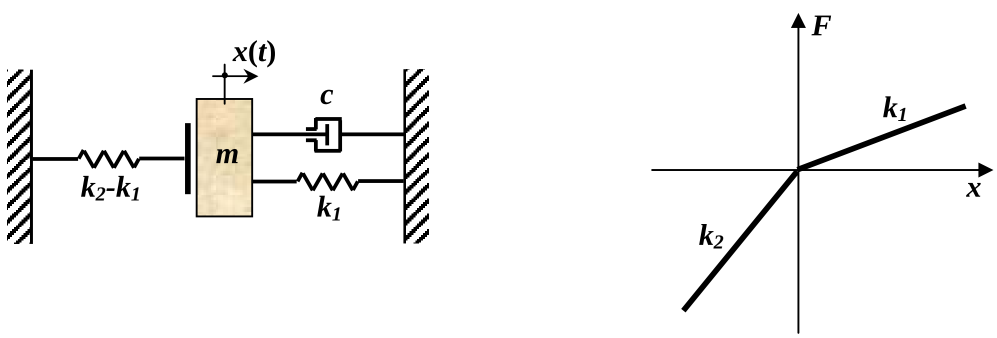
In this section, a novel system identification method is proposed for bilinear systems by separating the impulse or free-vibration responses at the stiffness interface into local responses.

Dynamic characteristics of bilinear systems

Consider a Single-Degree-Of-Freedom (SDOF) bilinear oscillator in which the change in stiffness occurs at its static equilibrium $x=0$, as shown in Fig. 1. Physically, each time the oscillator crosses the interface between the two stiffness regions ($x=0$), the stiffness of the system changes. The equation of motion of the system under free vibration can be expressed as

$$\begin{cases} m\ddot{x}+c_1\dot{x}+k_1x=0 & x\geq 0 \\ m\ddot{x}+c_2\dot{x}+k_2x=0 & x<0 \end{cases} \tag{1}$$

where x is the displacement of the bilinear oscillator; m and c are its mass and damping coefficients, respectively; k_1 and k_2 are the stiffness coefficients in the two stiffness regions.



(a) physical model of a bilinear oscillator (b) restoring force model of the bilinear oscillator

Figure 1. Physical model and stiffness characteristic of a bilinear oscillator

Nondimensionalizing Eq. (1) with mass m yields

$$\ddot{x}+2\xi_1\omega_1\dot{x}+\omega_1^2x=0 \quad x\geq 0 \tag{2a}$$

$$\ddot{x}+2\xi_2\omega_2\dot{x}+\omega_2^2x=0 \quad x<0 \tag{2b}$$

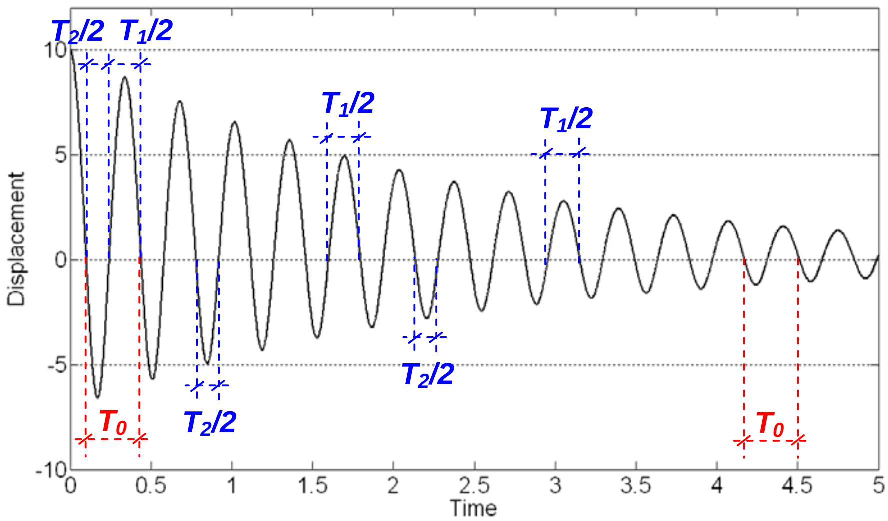
where ω_1 and ω_2 denote the circular frequencies of each stiffness region, respectively. Herein $\omega_1<\omega_2$. ξ_1 and ξ_2 are the modal damping ratios of each stiffness region, respectively.

Free-vibration responses of this bilinear oscillator can be obtained by solving Eq. (2a) and Eq. (2b) for responses in each of the two stiffness regions, and matching the responses in the two regions through the continuity of both displacement and velocity at the static equilibrium $x=0$, (Chu and Shen, 1992; Bayly, 1996). Figure 2 presents the free-vibration responses of the bilinear oscillator. From this figure, the free-vibration responses are periodical and the vibration period during the whole vibration duration is equal, designated T_0. The vibration period T_0 and vibration frequency ω_0 can be expressed as (Chu and Shen, 1992)

$$T_0 = \frac{T_1}{2} + \frac{T_2}{2} \quad \text{where } T_1 = \frac{2\pi}{\omega_1} \text{ and } T_2 = \frac{2\pi}{\omega_2} \tag{3}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\omega_1\omega_2}{\omega_1 + \omega_2} \tag{4}$$

Here, ω_0 is called the bilinear frequency (Chu and Shen, 1992), which represents the free-vibration frequency of the bilinear oscillator and satisfies $\omega_1 < \omega_0 < \omega_2$. Equation (4) holds rigorously true only for an undamped system. For a damped system, ω_1 and ω_2 should be replaced by the damped circular frequencies of each region. However, because the damping ratio is usually small in practice, Eq. (4) still holds true for a damped system.



Note: T_1 and T_2 represent the oscillation periods corresponding to the two regions, respectively

Figure 2. Displacement responses under free vibration of the bilinear oscillator

From Figure 2, although the vibration period of the system obtained from the global responses does not vary with time, the time durations of the two half sine waves in each period ($\frac{T_1}{2}$ and $\frac{T_2}{2}$, respectively) are different (to be exact, $\frac{T_1}{2} > \frac{T_2}{2}$, which is consistent with $\omega_1 < \omega_2$). However, the time durations of all half sine waves in the upper region (*displacement* >=0) are exactly the same and it is the same case for all half sine waves in the lower region (*displacement*<0). This suggests that although the system exhibits a nonlinear behavior in each entire vibration period, it still behaves linearly in each stiffness region.

Identification of natural frequency in each region

Actually, all positive responses are calculated from Eq. (2a) and all negative ones are calculated from Eq. (2b), and global responses are obtained by matching the two parts of local responses at the stiffness interface (*displacement*=0). Based on the above analysis, positive responses only include information on the parameters of the stiffness region $x \geq 0$ (parameters in Eq. (2a)), and negative responses only include information on the parameters of the stiffness region $x<0$ (parameters in Eq. (2b)). If the parameters of the region $x \geq 0$ are to be determined, only the positive responses are needed, and vice versa. Based on this observation, a novel, simple method is proposed to identify the natural frequency of each region.

First, the measured global impulse or free-vibration responses are divided into two parts at the stiffness interface (*displacement*=0 here). Figure 3 presents the separated responses above and below the axis of *displacement*=0. This simulates that only a half sine wave in each vibration period is acquired during the measurement.

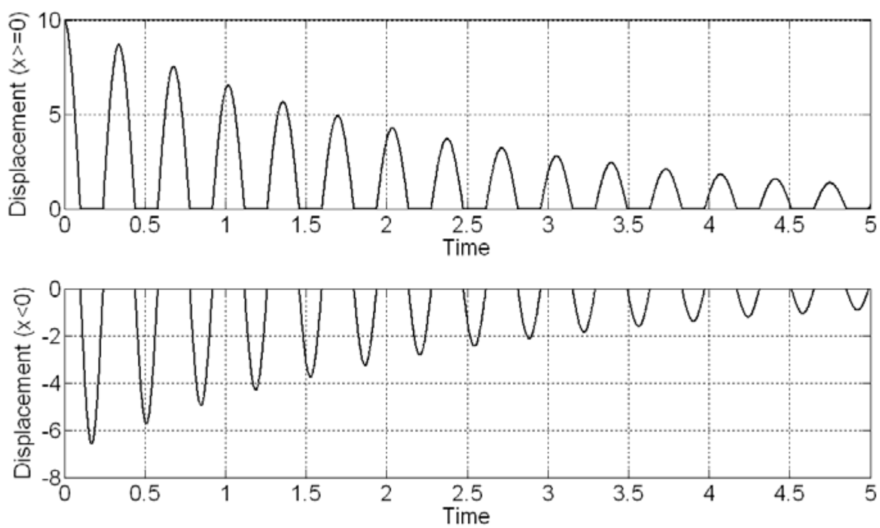
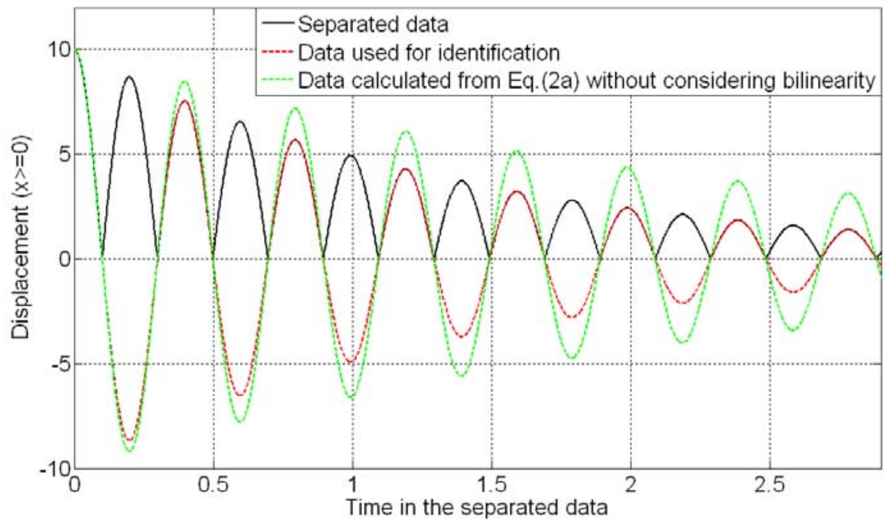


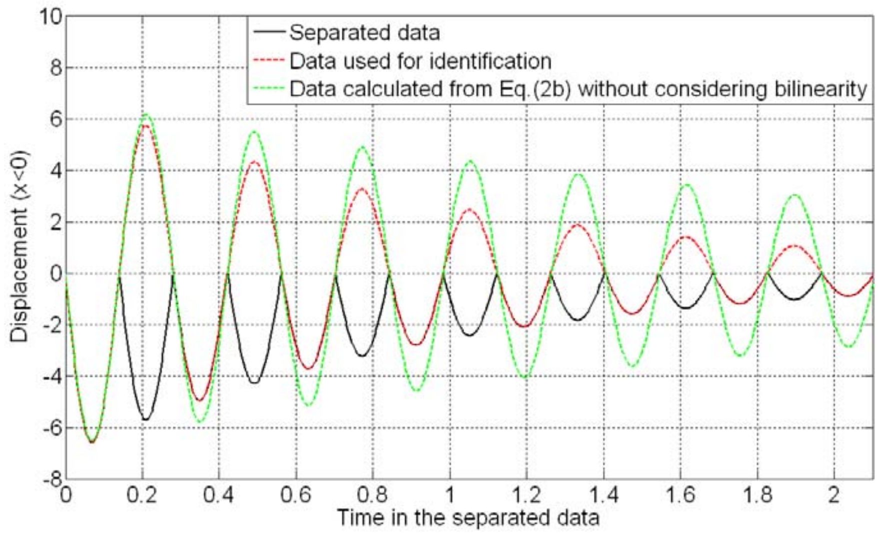
Figure 3. Separation of global responses into two parts at the stiffness interface (*displacement*=0)

Then, the half sine waves in each region are assembled together so as to reconstruct a signal in which only the half sine waves in the respective region are involved (the black graph in Fig. 4). In this paper, the assembled signal is called the local responses corresponding to each region, “local responses” for short. Because the period of the half sine wave in each set of local responses does not vary with time, Fourier transform or any other linear modal identification approach can be applied to identify modal parameters in the respective region. System parameters associated with the region $x \geq 0$ can be extracted from the local responses in the upper region (above the axis of *displacement*=0), and the system parameters associated with the region $x<0$ can be extracted from the local responses in the lower region (below the axis of *displaecement*=0). By separating global responses into two parts, the problem of nonlinear system identification is transferred to that of linear system identification. Actually, the frequency obtained from this assembled signal is twice the natural frequency of the associated region, and some higher harmonic components may occur.

To get the correct natural frequency of the associated region, an extra procedure may be performed on each set of local response data. That is, every second half sine wave is inverted to form entire sine waves (the red dashed graph in Fig. 4). Actually, this procedure generates a signal representing the responses of a linear system which has the same natural frequency as the system in each region, but has a different damping ratio from the system in each region, which can be reflected by the difference between the green and red graphs, as shown in Fig. 4. The identification of the damping ratio in each region is to be discussed in another paper. Compared with the nonlinear system identification methods reviewed by Farrar *et al.* (2007), the proposed method is much simpler.



(a) response data in region $x \geq 0$



(b) response data in region $x<0$

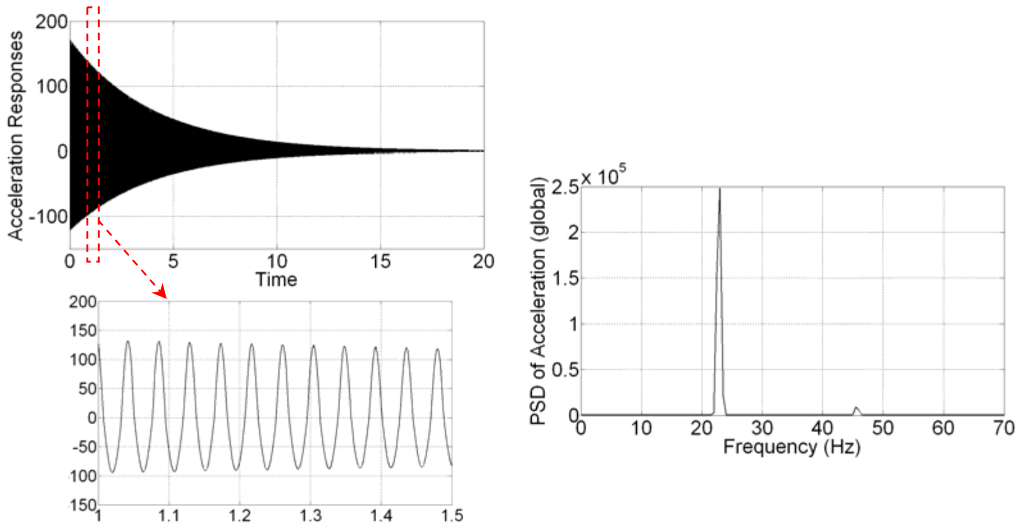
Figure 4. Generation of local response data for system identification

NUMERICAL SIMULATIONS

This example is to demonstrate the effectiveness of the proposed system identification method for bilinear systems. Consider a bilinear oscillator whose motion was governed by Eq. (1). The system parameters were chosen as m=1 kg, $k_2=3\times10^4$ Nm^{-1}, and $k_1 = \alpha k_2$. Here α represented the stiffness ratio and it was equal to 0.5 in this case. The natural frequencies in the regions $x \geq 0$ and x<0 were 19.49 Hz and 27.57 Hz, respectively. Assume that the damping ratios in the two regions were 0.14% and 0.20%, respectively.

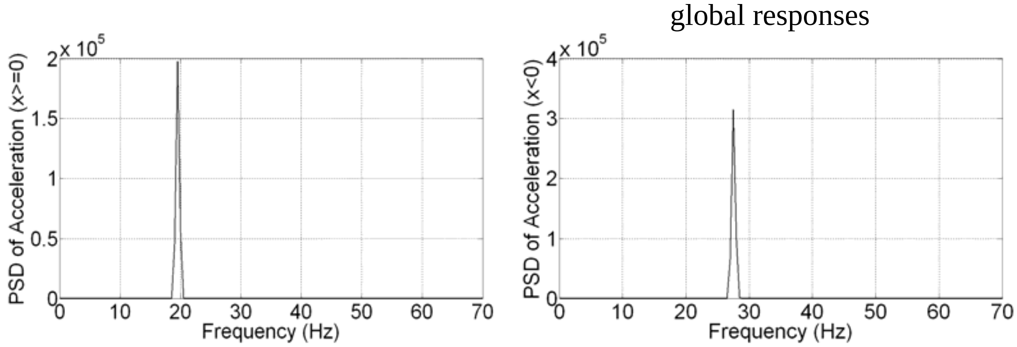
A numerical algorithm based on a fourth-order Runge-Kutta integration scheme was employed to calculate acceleration responses. To simulate an impact excitation, the initial displacement was set to 0 and the initial velocity was set to 1 *m/s*. The response time history was assumed to be acquired at the sampling frequency of 512 Hz and the acquisition duration was 20 seconds. To investigate the effectiveness of the proposed method in the presence of measurement noises, Gaussian white noises with the mean value of zero and the RMS (root-mean-square) equal to 5% and 10% of that of responses were added to the acceleration responses.

Figure 5a) plots the calculated acceleration response data. It is found that the response amplitudes are not symmetric about the axis of *displacement*=0, which is a time-domain distortion caused by bilinearity. In the auto power spectral density of global acceleration responses shown in Figure 5b), the peaks correspond to the bilinear frequency (23 Hz) and one higher harmonic component (46 Hz), which is the consequence of the presence of bilinearity.



a) acceleration time history

b) power spectral density (PSD) of global responses



c) PSD of local responses associated with the region $x \geq 0$

d) PSD of local responses associated with the region x<0

Figure 5. Acceleration responses and auto power spectral density of global and local response data

The procedures of the proposed method are as follows: 1) the global acceleration responses are first separated into two parts, forming two sets of local response data. Positive acceleration responses correspond to the region *x*<0, and vice versa. This corresponding relationship exists because the phase difference between the acceleration and displacement responses is π; 2) Among each set of local response data, through finding the transition points whose slopes change signs, each second half sine waves were inverted. For example, for the acceleration responses in the region x<0 (positive acceleration responses), the transition points were those whose slope signs change from *x*<0 to *x*>0; 3) the auto power spectral density associated with each region was obtained by performing Fourier transform on each set of local response data, as shown in Figure 5 (c) and (d). The only peak is associated with the natural frequency of the associated region. The identified natural frequencies are listed in Table 1. In this table, "Analytical" results denote results obtained by performing eigenvalue decomposition of system mass and stiffness matrices, and "Identified" results denote results extracted from simulated acceleration responses using the proposed method.

Table 1. Identified natural frequency of the bilinear oscillator (Hz)

	Analytical	Identified results		
		without noise	5% noise	10% noise
From global responses	22.84	22.84	22.84	22.84
From the responses x>0	19.49	19.49	19.45	19.45
From the responses x<0	27.57	27.47	27.65	27.78

From Table 1, it can be observed that the identification accuracy is all very high under different measurement noise levels using the proposed method. However, although the identified accuracy of the bilinear frequency from the global responses is also high, the natural frequency of each region cannot be obtained from the identified bilinear frequency.

CONCLUSIONS

To develop a simple but effective approach to detect breathing-fatigue cracks, a system identification method for bilinear systems was proposed by separating global impulse or free-vibration responses of the system into local responses corresponding to each stiffness region. This method transfers nonlinear system identification into linear system identification. Only either acceleration or displacement responses are required to be measured. The results from numerical simulations have demonstrated the effectiveness of the proposed method.

Further study will be conducted on: 1) validating the proposed method using experimental tests; 2) identifying the existence of breathing cracks and quantifying the cracks qualitatively by comparing the natural frequencies extracted from each set of local responses; 3) extending the proposed method to identify piecewise-nonlinear systems by introducing Hilbert transform.

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